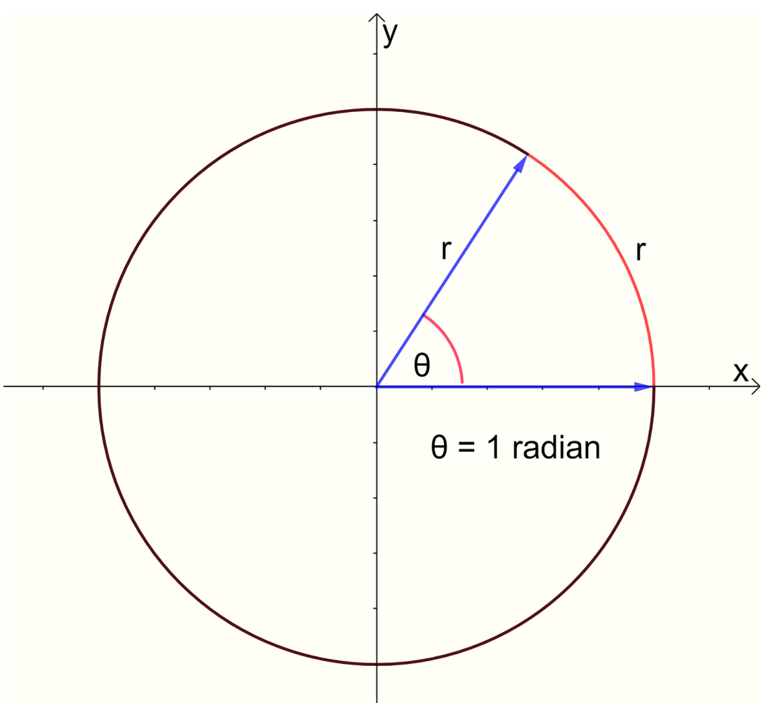
T5: Angular Measurement and the Unit Circle

Degrees	Radians
0	0
30	$\frac{\pi}{6}$
45	$\frac{\pi}{4}$
60	$\frac{\pi}{3}$
90	$\frac{\pi}{2}$
180	π
270	$\frac{3\pi}{2}$
360	2π

Angles are frequently measured in degrees. However it is sometimes useful to define angles in terms of the length around a circle. This module introduces radians as a measure of angle.

Definition of the Radian

Though angles have commonly been measured in degrees they may also be measured in units known as radians.
One radian is the angle created by bending the radius length around the arc of a circle. Consider the circle centred at the origin below. The red arc is the same length as the radius of the circle. The angle subtended at the origin θ, is one radian and is denoted by 1^c.



One radian is approximately 57.3°.

Converting Between Radians and Degrees

Because the circumference of a unit circle is given by the formula $C = 2\pi r$, we know 2π radians ($2\pi^c$) is a complete rotation and the same as 360 degrees. Similarly half a rotation or 180 degrees = π radians ($180^\circ = \pi^c$).

Angles that represent fractional parts of a circle can be expressed in terms of π.

Angle in Degrees	Angle in Radians
90	$\frac{\pi}{2}$
60	$\frac{\pi}{3}$
45	$\frac{\pi}{4}$
30	$\frac{\pi}{6}$
270	$3\times 90^\circ = 3 \times \frac{\pi}{2} = \frac{3\pi}{2}$

For other angles rearranging $\pi^c = 180^\circ$ gives:

$$1^c = \frac{180^\circ}{\pi}$$

and

$$1^\circ = \frac{\pi^c}{180}$$

Examples

1. Convert 60^0 to radians.

$$\begin{aligned}
1^\circ &= \frac{\pi^c}{180} \\
60^\circ &= 60 \times \frac{\pi^c}{180} \\
60^\circ &= \frac{60\pi^c}{180} \\
60^\circ &= \frac{\pi^c}{3} \\
60^\circ &\approx \frac{3.142^c}{3} \\
60^\circ &\approx 1.05^c
\end{aligned}$$

2. Convert 240^0 to radians.

$$1^\circ = \frac{\pi^c}{180}$$
$$240^\circ = 240 \times \frac{\pi^c}{180}$$
$$240^\circ = \frac{240\pi^c}{180}$$
$$240^\circ = \frac{4\pi^c}{3}$$

3. Convert $\frac{\pi}{4}$ radians to degrees.

$$1^c = \frac{180^\circ}{\pi}$$
$$\frac{\pi}{4}^c = \frac{\pi}{4} \times \frac{180^\circ}{\pi}$$
$$\frac{\pi}{4}^c = \frac{180^\circ}{4}$$
$$\frac{\pi}{4}^c = 45^\circ$$

4. Convert 6.5° to degrees.

$$1^c = \frac{180^\circ}{\pi}$$
$$6.5^c = 6.5 \times \frac{180^\circ}{\pi}$$
$$6.5^c \approx 372.4^\circ$$

Note: The symbol for radian, ^{c} , is often omitted.

Exercise

1. Convert the following degrees to radians
   a. 30° b. 270° c. 20° d. 450° e. 135° f. 57.3°.
2. Convert the following radians to degrees
   a. $\frac{\pi}{2}$ b. $\frac{5\pi}{4}$ c. $\frac{11\pi}{6}$ d. 3.5π e. π f. 1 radian.

Answers

1. a. $\frac{\pi}{6}$ b. $\frac{3\pi}{2}$ c. $\frac{\pi}{9}$
   d. $\frac{5\pi}{2}$ e. $\frac{3\pi}{4}$ f. 1 radian.
2. a. 90° b. 225° c. 330°
   d. 630° e. 180° f. 57.3°.