T2 Right Triangle Trigonometry

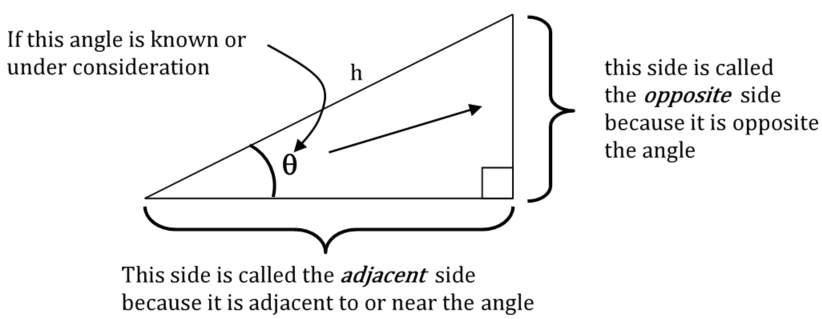
Trigonometry is a branch of mathematics involving the study of triangles, and has applications in fields such as engineering, surveying, navigation, optics, and electronics. The ability to use and manipulate trigonometric functions is necessary in other branches of mathematics, including calculus, vectors and complex numbers.

Right-angled Triangles

In a right-angled triangle the three sides are given special names.

The side opposite the right angle is called the hypotenuse (h) – this is always the longest side of the triangle.

The other two sides are named in relation to another known angle (or an unknown angle under consideration).



Trigonometric Ratios

In a right-angled triangle the following ratios are defined for a given angle θ

$$sine\,\theta = \frac{(opposite\ side\ length)}{(hypotenuse\ length)}$$
$$cosine\,\theta = \frac{(adjacent\ side\ length)}{(hypotenuse\ length)}$$
$$tangent\,\theta = \frac{(opposite\ side\ length)}{(adjacent\ side\ length)}$$

These ratios are abbreviated to $sin\,\theta$, $cos\,\theta$, and $tan\,\theta$ respectively.

A useful memory aid is SOH CAH TOA:

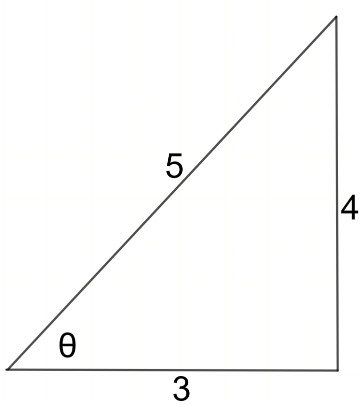
$$\mathbf{S}in\,\theta = \frac{\mathbf{O}pp}{\mathbf{H}yp} \qquad Cos\,\theta = \frac{Adj}{\mathbf{H}yp} \qquad \mathbf{T}an\,\theta = \frac{\mathbf{O}pp}{\mathbf{A}dj}$$

These ratios can be used to find unknown sides and angles in right-angled triangles.

Examples

Evaluating ratios

In the right-angled triangle below evaluate $sin\,\theta$, $cos\,\theta$, and $tan\,\theta$.

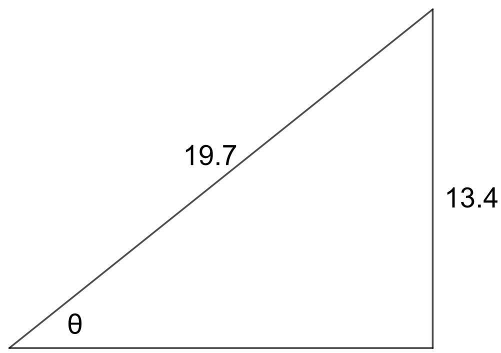


$$sin\theta = \frac{Opp}{Hyp} = \frac{4}{5} = 0.8$$
$$cos\ \theta = \frac{Adj}{Hyp} = \frac{3}{5} = 0.6$$
$$tan\ \theta = \frac{Opp}{Adj} = \frac{4}{3} \approx 1.33$$

Finding angles

Find the value of the angle in the triangle below[1]

[1] Step 1: Determine which ratio to use
Step 2: Write the relevant equation
Step 3: Substitute the values
Step 4: Solve the equation



In this triangle we know two sides and need to find the angle .

The known sides are the opposite side and the hypotenuse.

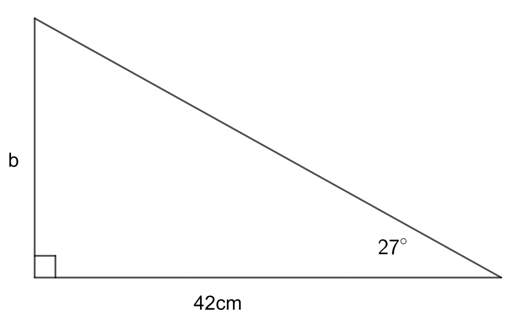
The ratio that relates the opposite side and the hypotenuse is the sine ratio.

$$\begin{aligned} sin\,\theta &= \frac{Opp}{Hyp} \\ sin\,\theta &= \frac{13.4}{19.7} \\ sin\,\theta &= 0.6802 \\ \theta &= sin^{-1}(0.6802) \\ \theta &= 42.9^{\circ} \end{aligned}$$

Finding side lengths

Find the value of the indicated unknown side length in each of the following right-angled triangles.

(a)



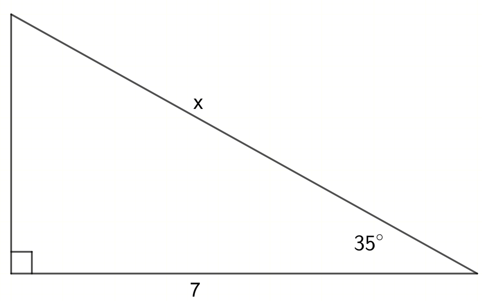
In this problem we know an angle and the adjacent side.

The side to be determined is the opposite side.

The ratio that relates these two sides is the tangent ratio.

$$\begin{aligned} tan\,\theta &= \frac{Opp}{Adj} \\ tan\,27^{\circ} &= \frac{b}{42} \\ (tan\,27^{\circ}) \times 42 &= b \\ b &= 21.4cm \end{aligned}$$

(b)



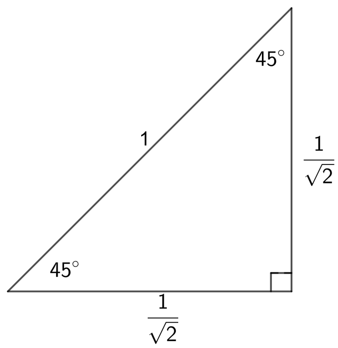
In this problem we know an angle and the adjacent side.
The side to be determined is the hypotenuse.
The ratio that relates these two sides is the cosine ratio.

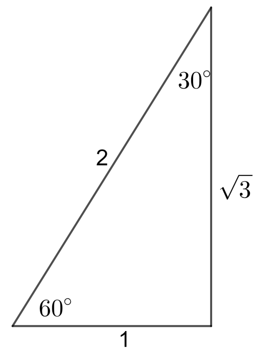
$$\begin{aligned} cos\,\theta &= \frac{Adj}{Hyp} \\ cos\,35^{\circ} &= \frac{7}{x} \\ (cos\,35^{\circ}) \times x &= 7 \\ x &= \frac{7}{cos\,35^{\circ}} \\ x &= 8.55 \end{aligned}$$

Special angles and exact values

There are some special angles for which the trigonometric functions have exact values rather than decimal approximations.

Applying the rules for sine, cosine and tangent to the triangles below, exact values for the sine, cosine and tangent of the angles 30°, 45° and 60° can be found.



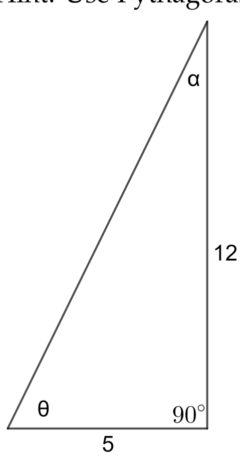


$$\sin 45^\circ = \frac{1}{\sqrt{2}} \qquad \cos 45^\circ = \frac{1}{\sqrt{2}} \qquad \tan 45^\circ = 1$$
$$\sin 60^\circ = \frac{\sqrt{3}}{2} \qquad \cos 60^\circ = \frac{1}{2} \qquad \tan 60^\circ = \sqrt{3}$$
$$\sin 30^\circ = \frac{1}{2} \qquad \cos 30^\circ = \frac{\sqrt{3}}{2} \qquad \tan 30^\circ = \frac{1}{\sqrt{3}}$$

Exercises

Exercise 1

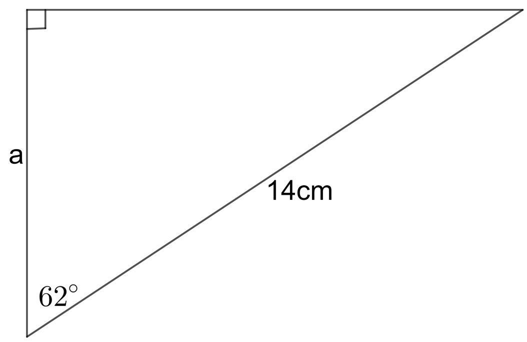
Using the right-angled triangle below find: (a) $\sin\theta$, (b) $\tan\theta$, (c) $\cos\alpha$, (d) $\tan\alpha$ (Hint: Use Pythagoras theorem to find the hypotenuse)



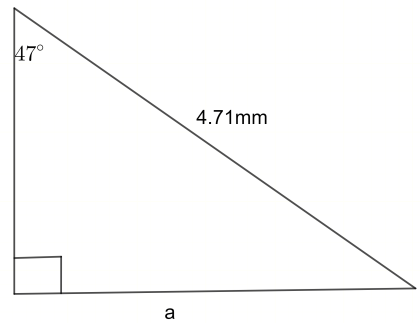
Exercise 2

Find the value of the indicated unknown (side length or angle) in each of the following diagrams.

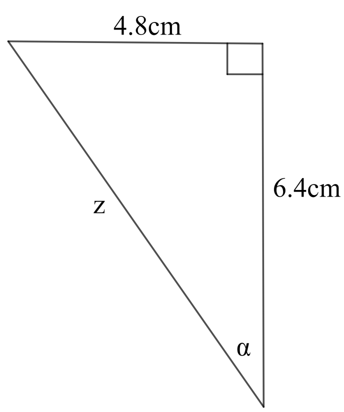
(a)



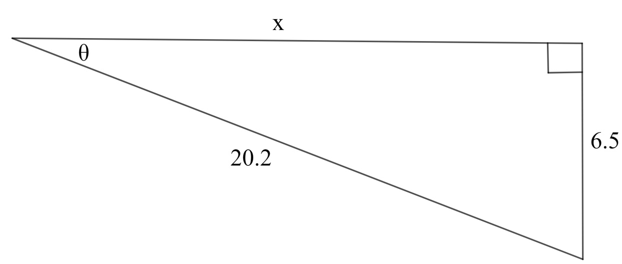
(b)



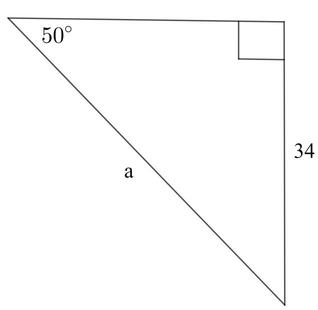
(c)



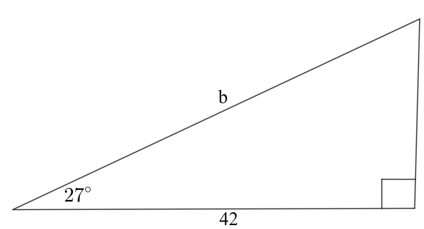
(d)



(e)



(f)



(g) In a right-angled triangle $sin\,\phi = 0.55$ and the hypotenuse is 21mm. Find the length of each of the other two sides.

Answers

Exercise 1
(a) $\frac{12}{13} = 0.9231$ (b) $\frac{12}{5} = 2.4$ (c) $\frac{12}{13} = 0.9231$ (d) $\frac{5}{12} = 0.4167$

Exercise 2
(a) = 6.6 cm (b) a = 3.4 mm (c) z = 7.8cm, $\alpha = 37.7^\circ$ (d) $\vartheta = 18.8^\circ$, x = 19.1 (e) a = 44.4 (f) b = 47.1 (g) 11.6mm. and 17.5mm.