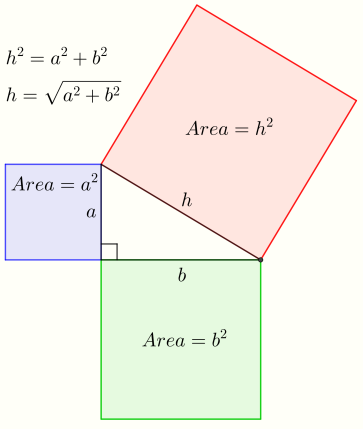
# T1: Pythagoras' Theorem




The Pythagorean theorem is a relationship between the lengths of the sides of a right angled triangle. It is useful for calculating side lengths in right triangles and is used in many parts of mathematics, science and engineering. It is one of those things you should know.

Play a short video.

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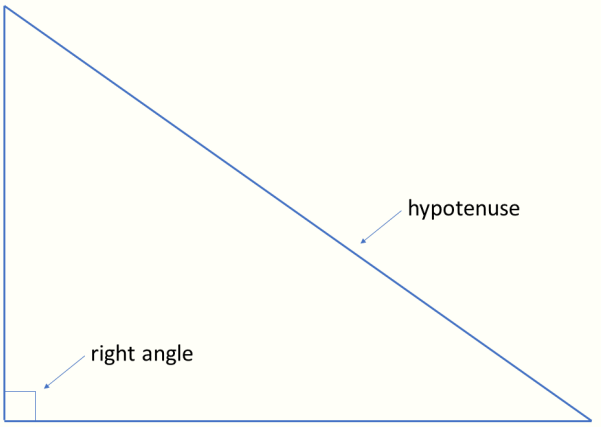
## *Right Angled Triangles*

Triangles are plane shapes with three straight sides. A right angled[1] triangle contains an angle of 90° as shown below:

[1] A right angle is an angle of 90°.

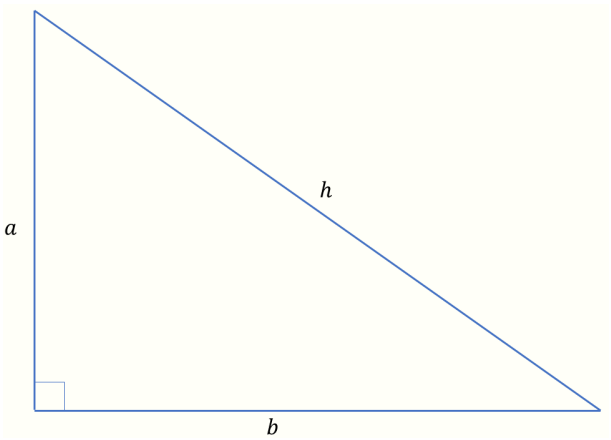



In a right angled triangle, the longest side is opposite the right angle and is called the hypotenuse.

## *Pythagoras' Theorem*

The theorem states that, in a right angled triangle[2], the square of the length of the hypotenuse is equal to the sum of the squares of the other two sides. In symbols:

[2] Pythagoras' Theorem does not apply to any triangle - only right angled triangles.

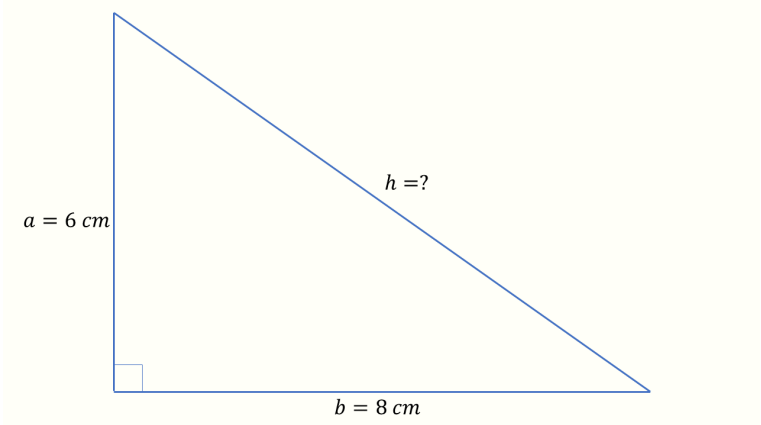



$$h^2 = a^2 + b^2.$$

Pythagoras' Theorem my be used to find the length of the third side of a triangle if you know the length of the other two sides.

*Example 1*

Find the length of the hypotenuse in the triangle below:




Solution: Using Pythagoras' Theorem we have:

$$\begin{aligned} h^2 &= a^2 + b^2 \\ &= 6^2 + 8^2 \\ &= 36 + 64 \\ &= 100. \end{aligned}$$

So the hypotenuse

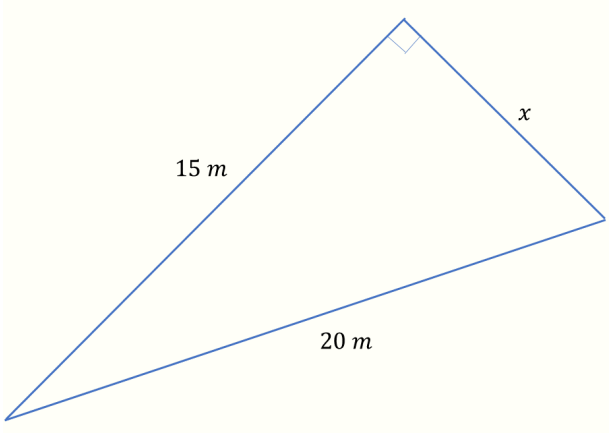
$$\begin{aligned} h &= \sqrt{100} \\ &= 10\,. \end{aligned}$$

The length[3] of the hypotenuse is $10\,cm$.

[3] Don't forget to include the unit of measurement in your answer.

*Example 2*

Find the length of the side $x$ to two decimal places in the triangle below:




Solution: In the case the hypotenuse $h = 20\,m$ and one of the shorter sides is $15\,m$. Using Pythagoras' Theorem we get:

$$20^2 = 15^2 + x^2.$$

Rearranging to make $x^2$ the subject we have:

$$\begin{aligned} x^2 &= 20^2 - 15^2 \\ &= 400 - 225 \\ &= 175. \end{aligned}$$

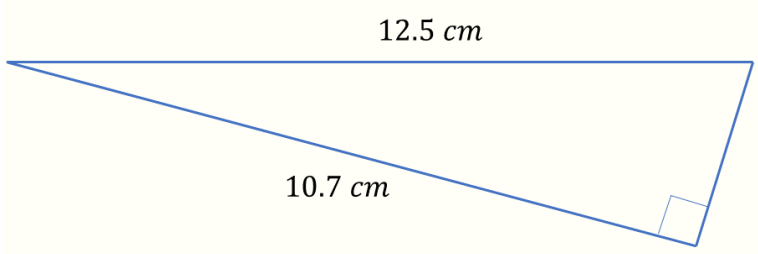
Taking the square root of both sides gives:

$$\begin{aligned} x &= \sqrt{175} \\ &= 13.23\,. \end{aligned}$$

The length of the side $x$ is $13.23\,m$.

*Example 3*

Find the length of the unknown side in the triangle below to two decimal places:




Solution: In this case the hypotenuse $h = 12.5\,cm$ and one of the shorter sides is $10.7\,cm$. Let the unknown side have length $x$ then

using Pythagoras' Theorem:

$$12.5^2 = 10.7^2 + x^2.$$

Rearranging to make $x^2$ the subject we have:

$$\begin{aligned} x^2 &= 12.5^2 - 10.7^2 \\ &= 156.25 - 114.49 \\ &= 41.76\,. \end{aligned}$$

Taking the square root of both sides gives:

$$\begin{aligned} x &= \sqrt{41.76} \\ &= 6.46\,. \end{aligned}$$

The length of the side is $6.46\,cm$.

## *Pythagorean Triples*

In some right angled triangles, the length of all three sides is an integer[4] in this case the three side lengths are called a Pythagorean Triple. Some examples are $(3,4,5)$, $(5,12,13)$, $(7,24,25)$ and $(8,15,17)$. Multiples of these numbers are also Pythagorean triples. Multiplying the triple $(3,4,5)$ by 2 gives a new triple $(6,8,10)$ as we saw in Example 1 above.

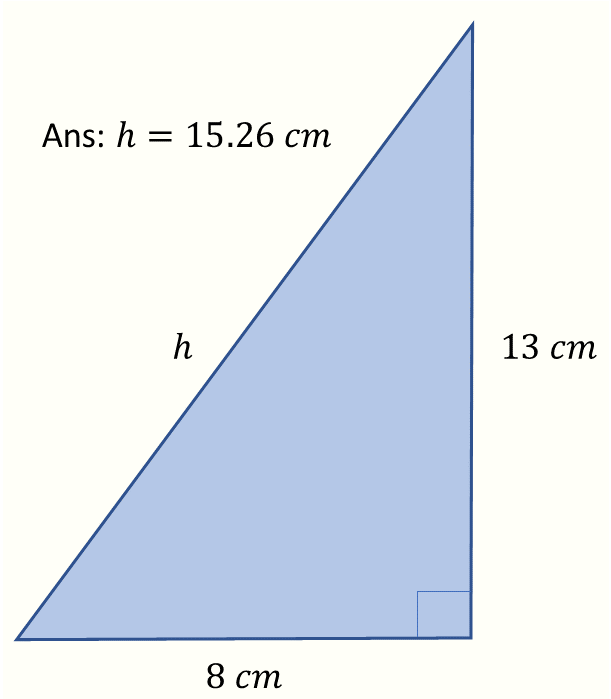
[4] The integers is the set of whole numbers and is denoted by

$$\mathbb{Z} = \{\cdots, -2, -1, 0, 1, 2, \cdots\}\,.$$

## *Exercises:*

Find the missing side length in the following triangles.

1.




2.