S18: Tests Of Proportion

A test of proportion is used to determine whether or not a sample from a population represents the true proportion from the entire population.

How to Perform a Test of Proportions

The steps to perform a test of proportion are:

1. Hypotheses
   State the null and alternative hypotheses. For example, if $\hat{p}$ is the sample proportion and p the null hypothesised population proportion, then for a 2-sided test:[1]

   $$H_o : \hat{p} = p$$

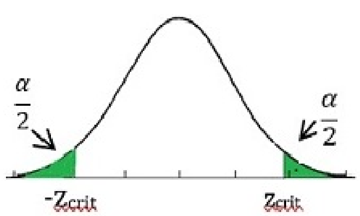
   (the proportion of the population from which the sample is drawn is the same as the given population proportion)

   $$H_a : \hat{p} \neq p$$

   (the proportion of the population from which the sample is drawn is not the same as the given population proportion)

2. A significance level α is chosen

3. z-tables are used to find the critical values



For the assumption that the test statistic is normally distributed to be valid we require $np > 5$ and $n(1-p) > 5$. (Check your course notes)

[1] Note: Some texts and courses may use other notation such as π to represent the population proportion, and p to represent the sample proportion.

4. Calculate the test statistic: $z = \frac{(\hat{p}-p)}{\sqrt{\frac{p(1-p)}{n}}}$

5. A decision is made regarding the "reasonableness" of the test statistic if H_o is true:
"Is the test statistic more extreme than the critical value?"
Yes $\Rightarrow$ Reject H_o
No $\Rightarrow$ Do not reject H_o

6. State your conclusion: There is (if you reject)/is not (if you do not reject) evidence to suggest that "....................". Paraphrase the words in the question to complete the sentence.

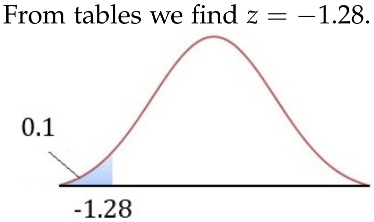
Example

A lecturer knows that historically 60% of students do not begin their statistics assignment until within seven days of the due date. As a result they do not do well. To address this unfortunate reality, maths support services are widely promoted during the early weeks of the semester. A survey of 80 randomly selected students revealed that 42 had started the assignment before the final week. Is there evidence to suggest the promotion has been effective at the 10% significance level?
In this case $\hat{p} = \frac{42}{80} = 0.525$.

1. Hypotheses:
$H_o : p = 0.6$ (the proportion of the population from which the sample is drawn is the same as the given population proportion)
$H_a : p < 0.6$ (the proportion of the population from which the sample is drawn is less than the given population proportion)

2. Significance level
$\alpha = 0.10$

3. Critical values
Assumption of normality is satisfied: $np = 48 > 5, n(1-p) = 32 > 5$
From tables we find $z = -1.28$.



4. Test statistic: $z = \frac{\hat{p}-p}{\sqrt{\frac{p(1-p)}{n}}} = \frac{0.525-0.6}{\sqrt{\frac{0.6(1-0.6)}{80}}} = -1.37$

5. -1.37 is more extreme than -1.28, therefore we reject H_o .

6. There is evidence to suggest that the promotion has been effective and that students are beginning their assignments earlier.

Exercises

1. A survey conducted ten years ago showed that 18% of students were the first in their family to attend a university. To evaluate widening participation programs a randomly selected sample of 120 enrolment records indicated that in 2017 this number had increased to 24%. Do these data provide convincing evidence to suggest that the percentage of 'first in family' students has changed over the last five years? Use $\alpha = 0.10$.
   Answer: $z = 1.71 \Rightarrow$ reject H_o
2. A car manufacturer advertises that at least 80% of vehicles will obtain fuel economy of $7\,l/100\,km$ or better. To test the claim, a consumer interest group randomly selects 22 vehicles and finds that only 15 achieve this figure. Is there evidence that the companies claim is false at the 5% level of significance?
   Answer: $z = -1.41 \Rightarrow$ do not reject H_o.
3. In a rural community hospital 6 out of 7 babies born in a particular week were boys. Assuming that in the general population the probability of female and male babies is equal, test the claim that babies born at this hospital are more likely to be boys at the 10% level of significance.
   Answer: $z = 1.90 \Rightarrow$ reject H_o(!). Note: Assumption of normality is NOT met .

Emoticon in title graphic is from Pixabay: https://pixabay.com/illustrations/smiley-yellow-happy-smile-emoticon-163510/