S16. *P-Value*

Introduction

In hypothesis testing two approaches are possible when making the decision as to whether to reject the null hypothesis. So far we have compared the test statistic with the critical value(s) and asked the question:

"Is the test statistic more extreme than the critical value?" Yes $\Rightarrow$ Reject H_o and No $\Rightarrow$ Do not reject H_o.

An alternative approach involves calculating the p-value and comparing it with α. The p-value is a probability. It is a measure of the likelihood of the test statistic being obtained if the null hypothesis is true. If that probability is low enough we reject the null hypothesis. How low is low enough? This is the value α and it is set before we calculate the test statistic. Our question becomes:

"Is p $< \alpha$?" Yes $\Rightarrow$ Reject H_o and No $\Rightarrow$ Do not reject H_o.
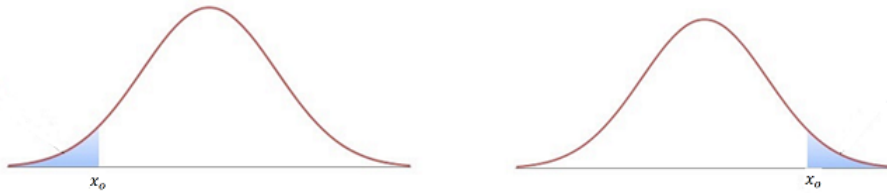
Calculation of the p-value

For a 2-sided test, where x_o is the test statistic, the p-value is the sum of the areas of the two regions beyond $-|x_o|$ and $|x_o|$ as shown below.
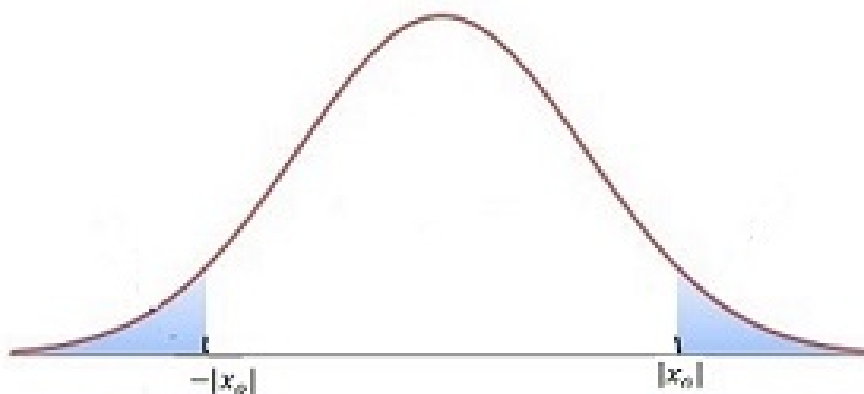
For a 1-sided test, where x_o is the test statistic, the p-value is either

the area to the left of x_o for a left handed test, or the area to the right of x_o for a right sided test as shown below.

The six steps to conduct a hypothesis test then become:

1. State the null and alternative hypotheses

2. Choose the level of significance, α

3. Calculate the test statistic

4. Compute the p-value of the test statistic

5. Decide whether to reject the null hypothesis by comparing the p-value to α

6. State your conclusion in words.

Example

The student queries service at a university claim that during orientation week the average wait time to be seen is 23 minutes. However this has been disputed as feedback from student focus groups indicates the wait is much longer. To test the hypothesis, the wait time for 40 randomly selected students is recorded during the next orientation week. The average wait time is found to be 25 minutes with a standard deviation of 5 minutes. Is there evidence to support the hypothesis that the average wait time exceeds 23 minutes at a 5% level of significance?

1. Hypotheses:
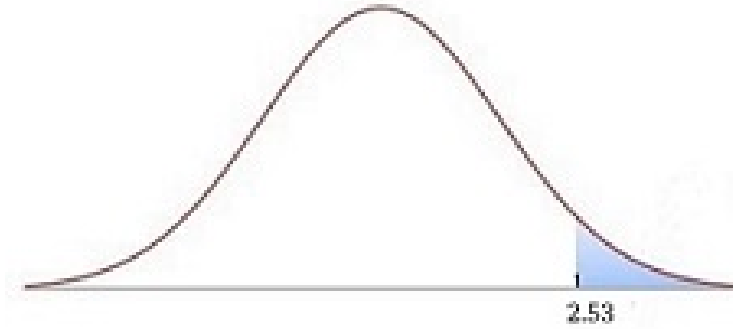 $H_o : \mu = 23$
   $H_a : \mu > 23$

2. Significance level
   $\alpha = 0.05$.

3. Test statistic[1]
   $t = \frac{\bar{x}-\mu}{s_{\bar{x}}} = \frac{25-23}{\frac{5}{\sqrt{40}}} = 2.53.$

[1] We are using a t-test because the population standard deviation is unknown.

4. P-value

2.53

The area to the right of 2.53 is 0.00778. [2]

5. Decision

Is $0.0078 < 0.05$? Yes, therefore we reject H_o.

6. Conclusion

There is evidence to suggest that the average waiting time for student queries during orientation week is more than 23 minutes.

Exercise

Your answers should be set out and contain all the steps required by your course. A brief outline of the main features is given in the answers below.

1. A sample of 10 is drawn from a normally distributed population and the mean is found to be 37.22. Use the p-value method to see if the sample mean is significantly less than the population mean of 40 if the population standard deviation is known to be 3.27 (use $\alpha = 0.05$).
   Answer: Test statistic $z = -2.69$, $p = 0.0037$. Reject H_o.

2. It is believed that traditionally 35% of RMIT students are aware of the SLC Maths Drop-in service. Management have recently implemented a new marketing program and results of a new survey show that out of the100 randomly sampled students 42 knew of the service. Is there evidence to suggest that the proportion of students at RMIT who are aware of the Maths Drop-in service has changed after the implementation of the marketing initiative? Use $\alpha = 0.1$ with the p-value method.
   Answer: Test statistic $z = \pm 1.47$, $p = 0.142$. Do not reject H_o.

3. The manufacturer of 'branbix' cereal claims the mean weight of contents of each box is 450g. A consumer interest magazine weighs a sample of 100 boxes and finds that they have a mean of 444g with a standard deviation of 28g. Use the p-value method to

decide whether the sample data contradict the manufacturer's and the boxes are being under filled? (use $\alpha = 0.01$)

Answer: Test statistic $t = -2.14$, $p = 0.017$. Do not reject H_o.