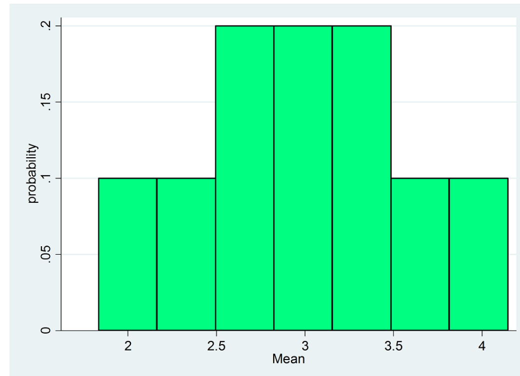
S12. Sampling Distributions



Introduction

A sampling distribution is the probability distribution for the means of all samples of size n from a given population.

The sampling distribution will be normally distributed with parameters $\mu_{\overline{x}}$ and $\sigma_{\overline{x}}$, if either:
(a) the population from which the samples are drawn is normally distributed, or
(b) the samples are large $(n \geqq 30)$.

The mean of the sampling distribution (i.e. the mean of all the sample means, $\mu_{\overline{x}}$) and the standard deviation of the distribution ($\sigma_{\overline{x}}$) are given by:

$$\mu_{\overline{x}} = \mu \quad \textit{and} \quad \sigma_{\overline{x}} = \frac{\sigma}{\sqrt{n}}$$

Note that:

- The sampling distribution has the same centre as the population.
- The measure of variability of a sampling distribution, $\sigma_{\overline{x}}$, is called the standard error.
- The distribution of means is not as spread out as the values in the population from which the sample was drawn.
- If we do not know the population standard deviation we approximate with the sample standard deviation: $s_{\overline{x}} \approx \sigma_{\overline{x}}$ and $\dfrac{s}{\sqrt{n}} \approx \dfrac{\sigma}{\sqrt{n}}$ if the sample is large.

Example of a Sampling Distribution

Consider the little 'population' of values P = {1 2 3 4 5}
This population has $\mu = 3$ and $\sigma = 1.41$.
If a sample of size n = 3 was drawn from this population it could be any one of:

(1 2 3) (1 2 4) (1 2 5) (1 3 4) (1 3 5) (1 4 5) (2 3 4) (2 3 5)
(2 4 5) (3 4 5)

The means of each of the samples, and a histogram of the distribution of means, are shown in the table and graph below:

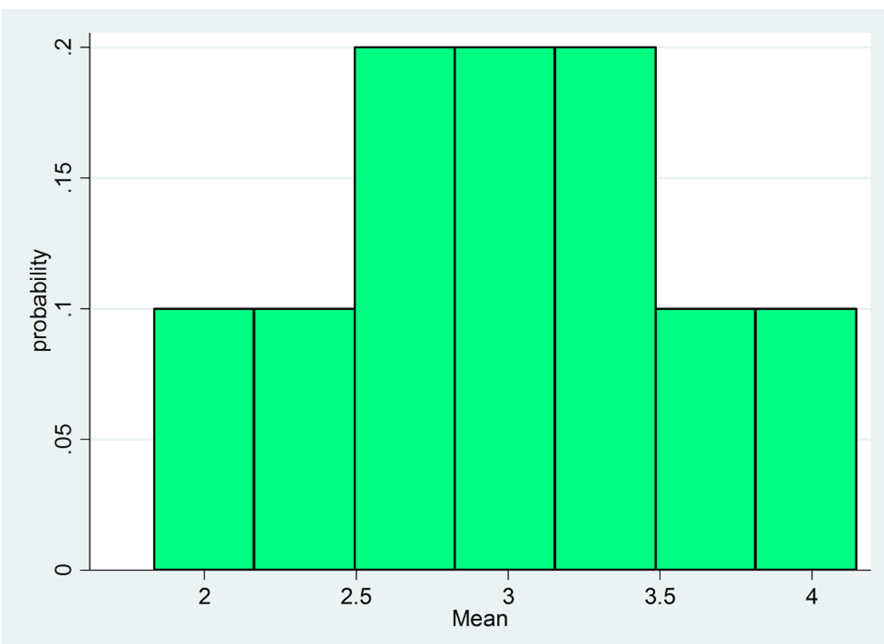
Sample	Mean
1 2 3	$\overline{x} = 2$
1 2 4	$\overline{x} = 2.33$
1 2 5	$\overline{x} = 2.67$
1 3 4	$\overline{x} = 2.67$
1 3 5	$\overline{x} = 3$
1 4 5	$\overline{x} = 3.33$
2 3 4	$\overline{x} = 3$
2 3 5	$\overline{x} = 3.33$
2 4 5	$\overline{x} = 3.67$
3 4 5	$\overline{x} = 4$

$\overline{\overline{x}} = 3$ and $\sigma_{\overline{x}} = 0.61$

The sampling distribution of the means for samples of size 3 is:

$\overline{x}$	2	2.33	2.67	3	3.33	3.67	4
$P(\overline{X} = \overline{x})$	0.1	0.1	0.2	0.2	0.2	0.1	0.1

Even though this sample is small, and the population is not normally distributed (though it is symmetric) the sampling distribution is reasonably normally distributed:



We can see that the mean of the sampling distribution (the mean of all the means) is the same as the population mean, $\overline{\overline{x}} = \mu = 3$. But the variability in the sampling distribution is less than that of the population: $\sigma_{\overline{x}} = 0.61$ and $\sigma = 1.41$. Because larger samples, or those drawn from normally distributed populations, will follow a normal distribution we can use the properties of normal distributions to find probabilities relating to samples:

$$z_{\overline{x}} = \frac{(\overline{x} - \mu)}{\sigma_{\overline{x}}} = \frac{(\overline{x} - \mu)}{\sigma/\sqrt{n}}.$$

Another Example

The shire of Bondara has 1200 preschoolers. The mean weight of pre-schoolers is known to be 18kg with a standard deviation of 3kg. What is the probability that a random sample of 50 preschoolers will have a mean weight more than 19kg?

$n = 50$, $\mu = 18$ and $\sigma = 3$

The sampling distribution of the means for samples of size 50 will have $\mu_{\overline{x}} = \mu = 18$, and standard error,

$$\sigma_{\overline{x}} = \frac{\sigma}{\sqrt{n}} = \frac{3}{\sqrt{50}} = 0.42$$

.

$$\begin{aligned} z_{\overline{x}} &= \frac{(\overline{x} - \mu)}{\sigma/\sqrt{n}} \\ &= \frac{(19 - 18)}{3/\sqrt{50}} \\ &= 2.38 \end{aligned}$$

$$\begin{aligned} Pr(\overline{x} > 19) &= Pr(z_{\overline{x}} > 2.38) \\ &= 1 - 0.9913 \qquad [from\ tables] \\ &= 0.0087 \end{aligned}$$

Exercise

1. List all samples of size 2 for the population $1, 2, 3, 4, 5, 6$. What is the probability of obtaining a sample mean of less than 3?

Answer: $4/15$

2. Samples of size 40 are drawn from a population with $\mu = 50$ and $\sigma = 5$. What are the mean and standard error of the sampling distribution? What is the probability that a particular sample has a mean less than 48.5?

Answer: (a) $\mu_{\overline{x}} = 50$ and $\sigma_{\overline{x}} = 0.79$ (b) 0.0288

3. If IQ in the general population of secondary students is known to follow a normal distribution with $\mu = 100$ and $\sigma = 10$, find the mean and standard error for a random sample of size 100. To test whether a secondary school is representative of the general population a sample of 100 students from that school is chosen. What is the probability of the mean IQ being more than 105? What would be your conclusion?

Answer: (a) $\mu_{\overline{x}} = 100$ and $\sigma_{\overline{x}} = 1$ (b) $0.00003 \approx 0$. This implies that either the sample was not random (perhaps all the smartest students were in the sample) or this school has a higher average IQ than the general population.