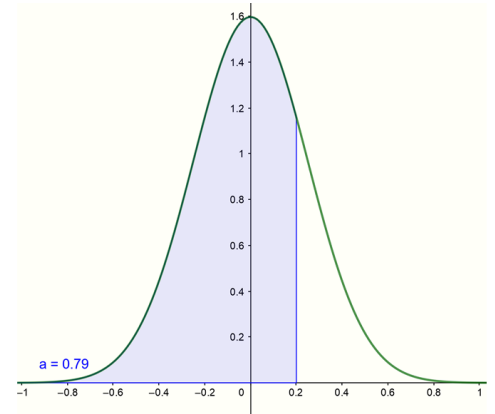
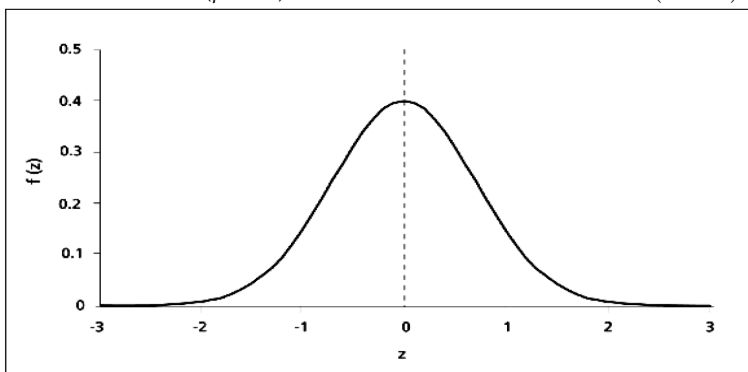


S10. Standard Normal Distribution

Introduction

The standard normal distribution (sometimes called a z-distribution) has a mean of zero ($\mu = 0$) and a standard deviation of 1 ($\sigma = 1$).



$$\Pr(X < 0.2) = 0.79$$

Tables and Probabilities

If we are working with the standard normal distribution we are not restricted to the 68-95-99 rule because tables are available to enable us to find proportions or percentages or probabilities for any value in the distribution. Tables come in different layouts, but this table (a portion of the table is shown below) gives the proportion to the left of a chosen z-value of up to 2 decimal places.

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7485
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7793
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078

We can interpret our proportions or percentages as probabilities:

$$\Pr(z < 0.00) = 0.500 \text{ or } 50\%.^1$$

$$\Pr(z < 0.03) = 0.512 \text{ or } 51.2\%.^2$$

$$\Pr(z < 0.75) = 0.7734 \text{ or } 77.34\%.^3$$

Note that a graphics calculator or a computer can also be used to find areas, proportions and probabilities in a normal distribution, rather than the table used here.

¹ (this corresponds to the number in line with 0.0 in the left hand column and 0.00 in the top row of the table)

² This corresponds to the number 0.0 in the left hand column and 0.03 in the top row of the table)

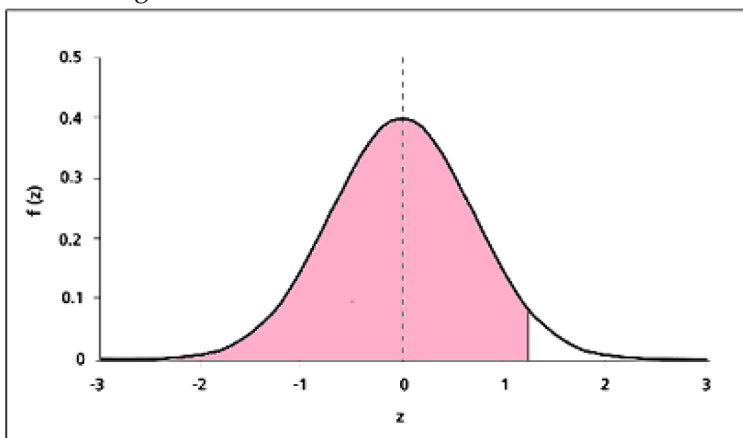
³ (this corresponds to the number in line with 0.7 in the left hand column and 0.05 in the top row of the table)

Example 1:

In a standard normal distribution what percentage of values will be less than 1.28?

Solution:

Draw a diagram as follows.



We are looking for the shaded area to the left of $z = 1.28$. Using the following table we find that the area to the left is 0.8997.

STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
1.0	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408

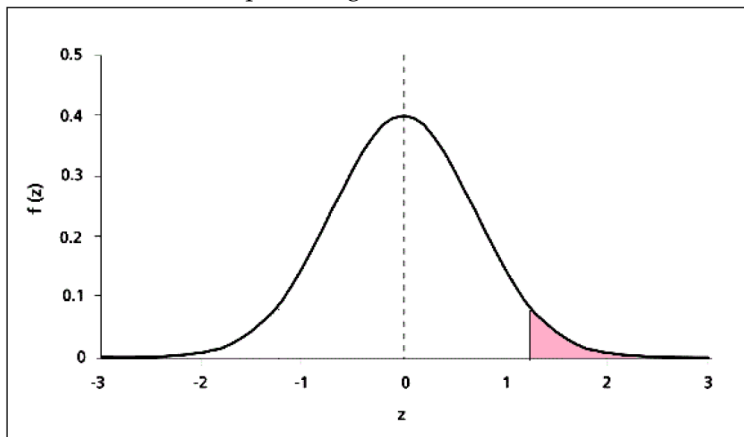
So the percentage is 89.97%.

Example 2:

In a standard normal distribution what percentage of values will be more than 1.28?

Solution:

We want to find the percentage area shown below.



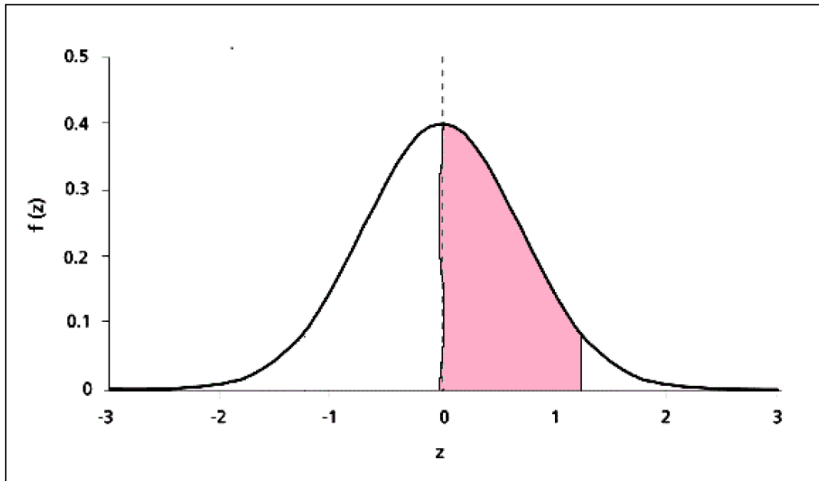
We cannot read areas to the right of a z-value directly from the table at the end of this module. Instead we observe that: 100% of all values lie under the curve. The area to the left of the shaded region is the same as part a). So the percentage of values to the right of 1.28 will be 100% minus the percentage to the left of 1.28. Therefore the percentage of values more than 1.28 is $100\% - 89.97\% = 10.03\%$.

Example 3:

In a standard normal distribution what percentage of values will be between 0 and 1.28?

Solution:

We want to find the percentage area shown below.



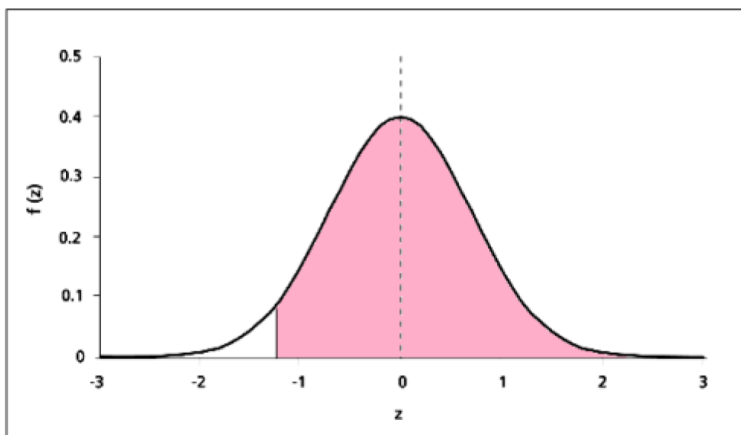
We cannot look up areas between two values directly from the table. But we know from Example 1 that 89.97% of values are less than 1.28. We also know 50% of values lie to the left of the mean ($z = 0$) because this is a property of our symmetrical bell curve. Therefore the percentage of values between 0 and 1.28 is $89.97\% - 50\% = 39.97\%$.

Example 4:

In a standard normal distribution what percentage of values will be greater than -1.28 ?

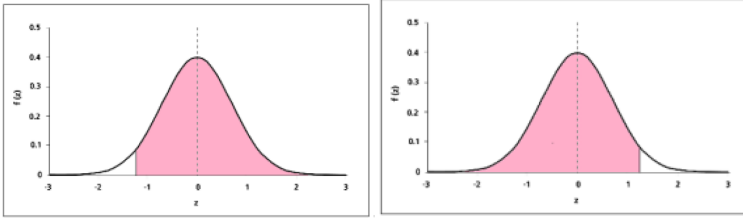
Solution:

We want to find the percentage area shown below.



We cannot usually look up negative values in the table⁴. But we know that the bell curve is symmetrical as shown below:

⁴ There are some normal distribution tables that include negative z values.



The area to the right of -1.28 is the same as the area to the left of 1.28 . Therefore the percentage of values greater than -1.28 is 89.97% from Example 1.

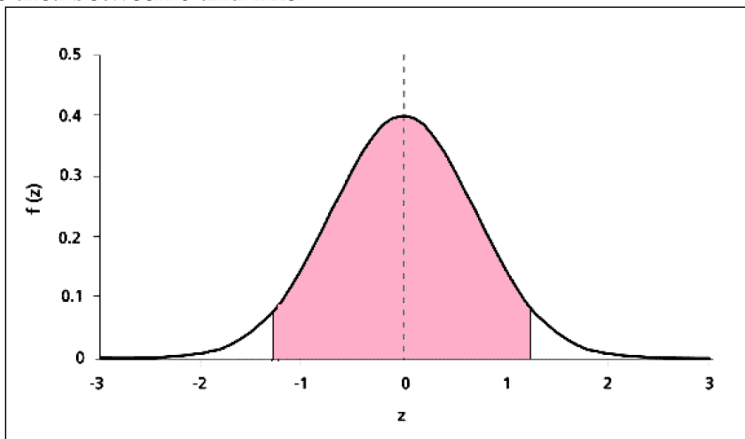
Example 5:

In a standard normal distribution what percentage of values will be between -1.28 and 1.28 ?

Solution:

To find this area we use the symmetry of the graph.

Observe that the area between -1.28 and 0 is exactly the same as the area between 0 and 1.28 .



To find this area we use the symmetry of the graph.

Observe that the area between -1.28 and 0 is exactly the same as the area between 0 and 1.28 .

We know the area between 0 and $1.28 = 39.97\%$ because we found it in Example 3 above.

Therefore the percentage of the graph between -1.28 and 1.28 is $2 \times 39.97\% = 79.8\%$.

Exercises

1. In a standard normal distribution what percentage of values will be ⁵

- less than 1.95 ?
- less than -1.95 ?

⁵ Note that answers may vary slightly depending on whether tables or calculators are used.

- (c) between -1.95 and 0 ?
- (d) greater than 1.95 ?
- (e) between -1.95 and 1.95 .

Answer: (a) 97.44% (b) 2.56% (c) 47.44% (d) 2.56 (e) 94.88%

2. In a standard normal distribution what proportion of values lie between -0.5 and 1.5 ?

Answer: 0.6247

3. In a standard normal distribution what proportion of values lie outside the interval ± 1.7 ?

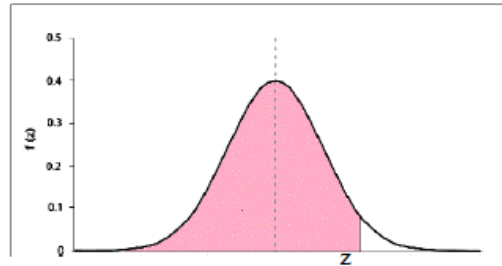
Answer: 0.0892

4. Given that a value in a standard normal distribution is greater than -1 what is the probability that it will be less than 2 ? (Hint: use the conditional probability formula.)

Answer: 0.9729

Table

The proportion of the area under the curve to the left of a chosen value of z is given in the table below.



Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9924	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9958	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986