SUMMER

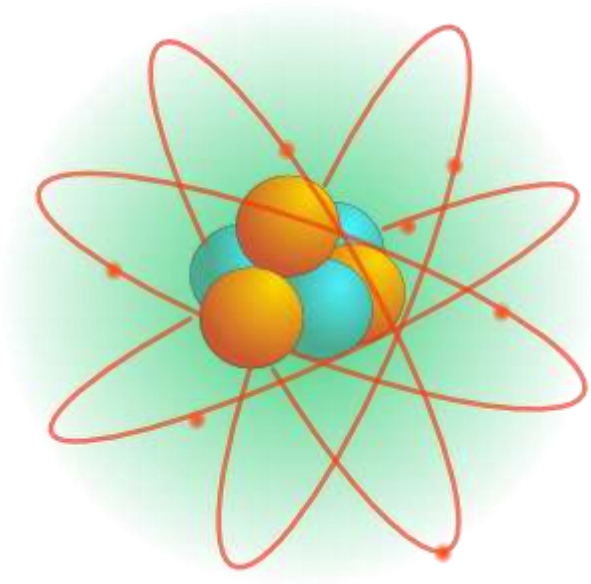
KNOWHOW

STUDY AND LEARNING CENTRE

RMIT UNIVERSITY

PHYSICS 1

PREPARATION FOR ENGINEERING, HEALTH, MEDICAL and APPLIED SCIENCE STUDENTS



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DIMENSIONS

In the SI system (metres, kilograms, seconds) the unit of speed is the metre per second. This is not the only unit that can be used. Some alternative, equally correct units are mile per hour, kilometre per hour, centimetre per minute, etc. All of these have one thing in common: each is a unit of length per unit of *time.*

The quantity length per time is called the **dimension** of the property speed.

When writing the dimensions of a physical quantity we use the symbols [L], [M], [T], and [I] to represent the dimensions of length, mass, time and current respectively. Hence we can write:

$$[\text{speed}] = [\text{length}]\,[\text{time}]^{-1} = [L]\,[T]^{-1}$$

where the brackets are read as 'dimensions of'.

Example 1

$$[\text{area}] = [L]^2 \qquad [\text{volume}] = [L]^3 \qquad [\text{density}] = [M]\,[L]^{-3}$$

The unit of all derived quantities can be found by substituting the fundamental unit for each dimension in the dimension equation. For example, in the SI system the unit of mass is the kilogram (kg) and the unit of length is the metre (m). The unit of density – dimension [M] $[L]^{-3}$ – then becomes kgm^{-3} in the SI system.

Checking Dimensions

The examination of the dimensions of an expression can be used to check the feasibility of given equations. The dimensions of both the left hand side and right hand side of all equations must be dimensionally correct and equal, so that they balance.

Example 2

The equation for the circumference of a circle:

$$C = 2\,\pi\,r$$

Left Hand Side: [circumference] = [L] a length

Right Hand Side: [2 π r] = [radius] = [L] ***Note:*** constant 2 π has no dimension. a length.

Hence the Left Hand side = Right Hand side, therefore the equation is dimensionally correct.

Exercise

Check that the following equations are dimensionally correct:

(a) $V = \pi r^2 h$ (the volume of a cylinder)

(b) $A = 4\pi r^2$ (the surface area of a sphere)

(c) $A = 2\pi r^2 + 2\pi rh$ (the surface area of a cylinder)

STUDY TIPS

SCIENTIFIC NOTATION

Scientific Notation (or Standard Form) is a way of writing numbers in a compact form. A number written in Scientific Notation is expressed as a number from 1 to less than 10, multiplied by a power of 10.

To write a number in Scientific Notation:

- move the decimal point so that there is **one** digit (which cannot be zero), **before** the decimal point.
- multiply by a power of 10, equal to the number of places the decimal point has been moved.

If the decimal point is moved to the **left**, the power of 10 is **positive**

If the decimal point is moved to the **right** the power of 10 is **negative**

Example 1
Write 5630 in Scientific Notation.

$5630 = 5.63 \times 1000 = 5.63 \times 10^3$ (remember: 1000 = 10^3)

Move the decimal point three places to the left. The number becomes 5.63.
The power of 10 is +3.

Example 2
Write 0.00725 in Scientific Notation.

$0.00725 = 7.25 \times 0.001 = 7.25 \times 10^{-3}$ (remember 0.001 = 10^{-3})

Move the decimal point three places to the right. The number becomes 7.25.
The power of 10 is –3.

Always check your answer.
If the magnitude of the number is less than one, then the power of 10 will be negative.

If the magnitude of the number is greater than or equal to 10 then the power of 10 will be positive.

Example 3

The mass of the moon is: 73 600 000 000 000 000 000 000 kg. = 7.36×10^{22} kg.

The charge on one electron is: 0.000 000 000 000 000 000 160 2 C = 1.602×10^{-19} C

Note how the size of the number is more easily seen when written in Scientific Notation. Errors are less likely when writing the number if it is in Scientific Notation.

Calculations can be simplified by using index laws. For example:

$3.5 \times 10^5 \times 5 \times 10^{18} = 3.5 \times 5 \times 10^{5+18} = 17.5 \times 10^{23} = 1.75 \times 10^{24}$

Exercise 1 Write the following numbers in Scientific Notation.

(a) 58000 (b) 0.0026 (c) 70.6 (d) 0.3 (e) 2 400 000 (f) 0.000 000 684

(g) 0.0704 (h) 0.260 (i) 17600 (j) 0.04080 (k) 0.000500 (l) 357 000 000 000

Exercise 2 Write the following numbers in Scientific Notation

(a) 6 370 000 m (the mean radius of the Earth)
(b) 86 400 s (the time for the Earth to orbit the sun)
(c) 0.0018 s (the half-life of a radioactive isotope)
(d) 380 000 000 m (the average distance of the moon form the Earth)
(e) 0.000 000 000 001 1 s (the time for a ray of light to pass through a window pane)
(f) 637 000 000 000 000 000 000 000 kg (the mass of the planet Mars)

Exercise 3 Write the following numbers in decimal form or in whole numbers.

(a) 7×10^2 (b) 4×10^0 (c) 3.46×10^3 (d) 5.96×10^{-5}

(e) 9×10^7 (f) 3.98×10^1 (g) 2.78×10^5 (h) 6.78×10^{-1}

Answers

Exercise 1

1. (a) 5.8×10^4 (b) 2.6×10^{-3} (c) 7.06×10 (d) 3×10^{-1} (e) 2.4×10^6 (f) 6.84×10^{-7} (g) 7.04×10^{-2} (h) 2.60×10^{-1}

(i) 1.76×10^4 (j) 4.08×10^{-2} (k) 5.00×10^{-4} (l) 3.57×10^{11}

Exercise 2

(a) 6.37×10^6m (b) 8.64×10^4s (c) 1.8×10^{-3}m (d) 3.8×10^8m (e) 1.1×10^{-12}s (f) 6.37×10^{23}kg

Exercise 3

(a) 700 (b) 4 (c) 3460 (d) 0.0000596

(e) 90000000 (f) 39.8 (g) 278000 (h) 0.678

STUDY TIPS

UNITS, PREFIXES, CONVERSIONS

In Physics we commonly use SI units (kilograms, metres, seconds) and prefixes, and we assume that you can convert between them. It is also assumed that you can give your answers in Scientific Notation. See table below.

Prefix	Symbol	Meaning	Example	Calculation with answer in Scientific Notation
pico	p	10^{-12}	15 picofarad	15 pF = (15×10^{-12})F = 1.5×10^{-11}F
nano	n	10^{-9}	35 nanosecond	35 ns = (35×10^{-9})s = 3.5×10^{-8}s
micro	μ	10^{-6}	52 microamp	52 μA = (52×10^{-6})A = 5.2×10^{-5}A
milli	m	10^{-3}	50 millivolt	50 mV = (50×10^{-3})V = 5.0×10^{-2}V
centi	c	10^{-2}	10 centimetre	10 cm = (10×10^{-2})m = 0.1m
kilo	k	10^{3}	3000 kilo ohm	3000 kΩ=(3000×1000) Ω=3.0×10^{6}Ω
mega	M	10^{6}	3000 megalitre	3000 Ml = (3000×10^{6})l = 3.0×10^{9}l
giga	G	10^{9}	125 gigawatt	125 GW=(125×10^{9})W=1.25×10^{11}W

Conversions between units

Example 1

Convert 1 km to millimetres

Step 1: Write down the relationship between km and mm
1 km = 1000 m and 1 m = 1000 mm
∴ 1 km = 1000 000 mm

Step 2: Convert to Scientific Notation
1 km = 1000 000 mm = 1.0×10^{6} mm

Example 2

Convert 0.3 gram to milligrams

Step 1: Write down the relationship between grams and milligrams
1 gram = 1000 milligrams = 1.0×10^{3} mg

Step 2: Write down the given quantity as a multiple
0.3 g = 0.3 × 1 g = 0.3×10^{3}mg

Step 3: Convert to Scientific Notation
0.3×10^{3}mg = 300 mg = 3.0×10^{2}mg

Example 3

Convert 1 μm to kilometres

Step 1: Write down the relationship between kilometres and μm <u>first</u>

1 km = 1000 m = 10^3m and 1 m = 10^6 μm

$\therefore$ 1 km = $10^3 \times 10^6 = 10^9$ μm

Step 2: **Now** write down the relationship between μm and km

1 km = 10^9 μm $\therefore$ 1 μm = 10^{-9}km

Step 3: Make sure the answer is in Scientific Notation (which in this case it is)

Exercise 1

Complete the following conversions, giving your answers in Scientific Notation.

1m	=	cm	1cm =		m
1m	=	mm	1mm =		m
1m	=	μm	1 μm =		m
1cm	=	mm	1mm =		cm
1km	=	m	1m	=	km
1km	=	cm	1cm =		km
1km	=	mm	1mm =		km
1kg	=	g	1g	=	kg
1kg	=	mg	1mg =		kg
1kg	=	μg	1 μg =		kg
1g	=	mg	1mg =		g
1g	=	μg	1 μg =		g
1mg	=	μg	1 μg =		mg

Other Conversions

To change from hours to seconds we multiply by 3600 since there are (60 $\times$ 60) seconds in 1 hour.

Conversely, we divide by 3600 to change from seconds to hours

Velocity may be given in km h^{-1}. To change to ms^{-1} involves two conversions:

1 km h^{-1} = 1000 m h^{-1} = (1000/3600) ms^{-1} = (1/3.6) ms^{-1}.

Example 4

60 kmh^{-1} = (60/3.6) = 16.7 ms^{-1}. To change from kmh^{-1} to ms^{-1} **divide** by 3.6

Example 5

20 ms^{-1} = 20 $\times$ 3.6 = 72 kmh^{-1}. To change from ms^{-1} to kmh^{-1} **multiply** by 3.6

Area:

1 m^2 = 10000 cm^2 = 10^4 cm^2; or 1cm^2 = 10^{-4} m^2

1 m^2 = 1000000 mm^2 = 10^6 mm^2; or 1mm^2 = 10^{-6} m^2

Volume

1 m^3 = 1000000 cm^3 = 10^6 cm^3; or 1cm^3 = 10^{-6} m^3

1 m^3 = 1000000000 mm^3 = 10^9 mm^3; or 1mm^3 = 10^{-9} m^3

Example 6

50 m^2 = 50 $\times$ 10^4 cm^2 = 5.0 $\times$ 10^5 cm^2. ***Note*** answer is in Scientific Notation

Example 7

68 mm^2 = 68 $\times$ 10^{-6} m^2 = 6.8 $\times$ 10^{-5} m^2 ***Note*** answer is in Scientific Notation

Example 8

450 m^3 = 450 $\times$ 10^6 cm^3 = 4.5 $\times$ 10^8 cm^3. ***Note*** answer is in Scientific Notation

Example 9

122 mm^3 = 122 $\times$ 10^{-9} m^3 = 1.22 $\times$ 10^{-7} m^3 ***Note*** answer is in Scientific Notation

Exercise 2

(a) Convert 7 cm^2 to mm^2

(b) Convert 3 $\times$ 10^3 m^2 to cm^2

(c) Convert 0.04 km^2 to m^2

(d) Convert 6 $\times$ 10^{-2} m^2 to cm^2

(e) Convert 80 mm^3 to cm^3

(f) Convert 30 m^3 to cm^3

(g) Convert 2 $\times$ 10^5 cm^3 to m^3

(h) Convert 0.06 mm^3 to m^3

(i) Convert 45 kmh^{-1} to ms^{-1}

(j) Convert 26 ms^{-1} to kmh^{-1}

(k) Convert 11 g cm^{-3} to kg m^{-3}

(l) Convert 5 cm^2 s^{-1} to m^2 h^{-1}

(m) Convert 8 kg m^{-2} to g cm^{-2}

(n) Convert 380 cm^3 s^{-1} to m^3h^{-1}

ANSWERS

Exercise 1

1m	=	10^2 cm	1cm =	10^{-2}m
1m	=	10^3 mm	1mm =	10^{-3}m
1m	=	10^6 μg	1 μm =	10^{-6}m
1cm	=	10 mm	1mm =	10^{-1} cm
1km	=	10^3 m	1m =	10^{-3}km
1km	=	10^5 cm	1cm =	10^{-5}km
1km	=	10^6 mm	1mm =	10^{-6}km
1kg	=	10^3 g	1g =	10^{-3}kg
1kg	=	10^6 mg	1mg =	10^{-6}kg
1kg	=	10^9 μg	1 μg =	10^{-9}kg
1g	=	10^3 mg	1mg =	10^{-3}g
1g	=	10^6 μg	1 μg =	10^{-6}g
1mg	=	10^3 μg	1 μg =	10^{-3}mg

Exercise 2

(a) 7×10^2 mm^2 (700 mm^2)

(b) 3×10^7 cm^2

(c) 4×10^4 m^2

(d) 6×10^2 cm^2

(e) 8×10^{-2} cm^3

(f) 3×10^7 cm^3

(g) 0.2 m^3

(h) 6×10^{-11} m^3

(i) Convert 12.5 ms^{-1}

(j) 93.6 kmh^{-1}

(k) 1.1×10^4 kg m^{-3}

(l) 1.8 m^2 h^{-1}

(m) 0.8 g cm^{-2}

(n) 1.36 m^{3}h^{-1}

STUDY TIPS

SIGNIFICANT FIGURES: RULES

In scientific measurements, significant figures are used to indicate the accuracy of a measurement. The last digit in a measurement is often an estimate, a good guess.

All the digits in a measured value, including the last estimated digit, are called significant figures or significant digits.

Consider a series of measurements made with (a) an old wooden ruler without mm marks, (b) a more accurate steel ruler, (c) steel callipers with a Vernier scale.

	Reported Result	Number of Significant Figures in the measurement	Accuracy of measurement	Implied range of possible values
(a)	7 cm	1	To the nearest 1 cm, ie 7 ± 0.5 cm	6.5 cm – 7.5 cm
(b)	7.2 cm	2	To the nearest 0.1 cm, ie 7.2 ± 0.05 cm	7.15 – 7.25 cm
(c)	7.23	3	To the nearest 0.01 cm, ie 7.23 ± 0.005 cm	7.225 – 7.235 cm

The number of Significant Figures in a measured value

Rule 1 Any non-zero digit is significant. The position of a decimal point makes no difference.

Example 1	15.7	3	sig. figs
	157	3	sig. figs
	1.57	3	sig. figs
	2.7942	5	sig. figs

Rule 2 Zeros between numbers **are** significant.

Example 2	1.05	3	sig. figs
	10.51	4	sig. figs
	200.708	6	sig. figs

Rule 3 Zeros at the right hand end of whole numbers are **not** significant, unless otherwise stated.

Example 3	70	1	sig. figs
	2860	3	sig. figs
	15090	4	sig. figs

Rule 4 Zeros at the left hand end of decimal numbers are **not** significant.

Example 4	0.28	2	sig. figs
	0.0039	2	sig. figs
	0.0604	3	sig. figs

Rule 5 Zeros at the right hand end of decimal numbers **are** significant.

Example 5	12. 0	3	sig. figs
	0.760	3	sig. figs
	0.48300	5	sig. figs
	2.07090	6	sig. figs

Significant Figures and Scientific Notation

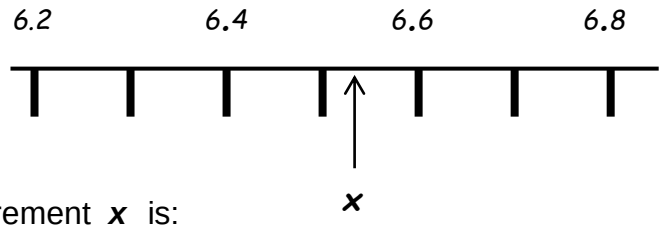
The problems of deciding how many significant figures a value has is simplified by writing its value in Scientific Notation.

Examine the number of digits in the first number below, not in the power of ten.

(a)	$0.0003 = 3 \times 10^{-4}$	1 sig fig	Rule 4
(b)	$720000 = 7.2 \times 10^{5}$	2 sig figs	Rule 3
(c)	$660 = 6.6 \times 10^{2}$	2 sig figs	Rule 3
(d)	$660.0 = 6.600 \times 10^{2}$	4 sig figs	Rule 5
(e)	$0.66000 = 6.6000 \times 10^{-1}$	5 sig figs	Rule 5
(f)	$808.01 = 8.0801 \times 10^{2}$	5 sig figs	Rule 2

Recording measurements and Significant Figures

Consider the scale below.



The measurement ***x*** is:

- Certainly greater than 6 and less then 7
- Certainly greater than 6.5 and less than 6.6
- Very probably greater than 6.53 and less than 6.55

The result would be recorded as 6.54 - a value with 3 significant digits. The first two digits are certain and the last is a good estimate.

Writing "6.54" implies "6.54 ± 0.005", ie between 6.535 and 6.545, unless otherwise stated.

Exercise

Write each of the following measurements in Scientific Notation and state the number of significant digits in the value.

(a)	345	(b)	17642	(c)	0.0033	(d)	0.000306
(e)	870	(f)	20000	(g)	140.600	(h)	710.0
(i)	0.04080	(j)	0.0050				

Answers

Exercise

(a)	345	3.45×10^{2}	3 sig figs
(b)	17642	1.7642×10^{4}	5 sig figs
(c)	0.0033	3.3×10^{-3}	2 sig figs
(d)	0.000306	3.06×10^{-4}	3 sig figs
(e)	870	8.7×10^{2}	2 sig figs
(f)	20000	2×10^{4}	1 sig fig
(g)	140.600	1.40600×10^{2}	6 sig figs
(h)	710.0	7.100×10^{2}	4 sig figs
(i)	0.04080	4.080×10^{-2}	4 sig figs
(j)	0.0050	5.0×10^{-3}	2 sig figs

SIGNIFICANT FIGURES: CALCULATIONS

Rounding Off

Sometimes, numbers must be "rounded off" so that values do not appear to have more significant figures than would be appropriate or realistic.

Rule 1 If the last digit is greater than 5, the second last digit increases by 1 when rounding off.

Example 5.78 rounded off to 2 sig figs becomes 5.8
5.8 rounded off to 1 sig fig becomes 6 (not 6.0)

Rule 2 If the last digit is less than 5, the second last digit does not change.

Example 1.304 rounded off to 3 sig figs becomes 1.30
1.30 rounded off to 2 sig figs becomes 1.3
1.3 rounded off to 1 sig fig becomes 1.

Rule 3 If the last digit is 5, the number is rounded off to the nearest even number.

Example 4.85 rounded off to 2 sig figs becomes 4.8
4.55 rounded off, becomes 4.6

Exercise 1

Round off each of the following numbers to one less significant figure and then to two less significant figures.

Example: 6.549 → 6.55 → 6.6

(a) 7.668 (b) 0.0854 (c) 21.092

(d) 255.6 (e) 35.3 (f) 6.75 (g) 2.85

Addition and Subtraction

When adding or subtracting, line up the values with the decimal places aligned. Examine the example below.

Only complete columns can be added or subtracted, so first round off all values to the same number of decimal places.

Example 1

Add 6.703, 2.49 and 11.7368, giving the answer to the correct number of decimal places.

6.703	rounds off to 2 dec. places	6. 70
2.49	rounds off to 2 dec. places	2. 49
11.7368	rounds off to 2 dec. places	11. 74
		20. 93

Answer: 20.93 (not 20.9298)

Note: If numbers are added and subtracted by the method shown above, the number of significant figures in the answer will automatically be correct.

Exercise 2

Calculate the following, giving answers to the correct number of decimal places.

(a) 6.9 + 0.35 + 12.625 (b) 27.1 + 13 + 64.51

(c) 8.72 – 3.001 – 0.2 (d) 67.4 – 18.37 + 0.55

(e) 0.003 + 0.0125 – 0.00378

Multiplication and Division

When multiplying and dividing, the answer must not have more significant figures than the original value with the least number of significant figures.

Example 2

$5.2 \times 6.3 = 33.76$ if you multiply it completely. However both the original values have only 2 sig figs, the answer must be rounded off to 2 sig figs.

Thus, $5.2 \times 6.3 = 33$ correct to 2 sig figs.

Example 3

$0.93 \times 5.41 = 5.03013$

If you multiply it all the way, but one of the original values has 2 sig figs, while the other has 3 sig figs, the anwer must be given to 2 sig figs, ie the anwer must not have more sig figs than the original value with the least number of sig figs.

Thus, $0.93 \times 5.41 = 5.0$, correct to 2 sig figs.

Example 4

$2.70 \div 16.44 = 0.1642335 = 0.164$,

$2.70 \div 16.44 = 0.1642335 = 0.164$, corrected to 3 sig figs, ie the answer must not have more sig figs than the original value with the least number of sig figs

Exercise 3

Calculate the following, giving answers to the correct number of significant figures.

(a) 8.3×0.25 (b) 0.2×1.3 (c) $1.13 \times 3.5 \times 0.964$

(d) 6.71×3.4 (e) $3.000 \times 91 \div 72.60$ (f) $450 \times 3 \div 0.671$

ANSWERS

Exercise 1

(a) 7.668 → 7.67 → 7.7
(b) 0.0854 → 0.085 → 0.08
(c) 21.092 → 21.09 → 21.1
(d) 255.6 → 256 → 260
(e) 35.3 → 35 → 40
(f) 6.75 → 6.8 → 7
(g) 2.85 → 2.8 → 3

Exercise 2

(a) 19.9 (b) 104 (c) 5.5 (d) 49.6 (e) 0.011

Exercise 3

(a) 2.1 2 sig figs
(b) 0.3 1 sig fig
(c) 3.8 2 sig figs
(d) 2.3×10^1 2 sig figs
(e) 3.8 2 sig figs
(f) 2×10^3 1 sig fig

ERRORS IN MEASUREMENT

Every measurement in science is an approximation. For various reasons, one can never be absolutely certain about the exact value of any measurement.

Mistakes

Mistakes include such things as:

(i) Writing down the wrong value, e.g. the scale reads “72” but you write down “27” by mistake.

(ii) Using the wrong pipette, e.g. adding 1 ml instead of 10 ml.

(iii) Misusing the calculator, e.g. pressing the “÷” button instead of the “×”.

In the scientific sense, the term mistake does not mean the same as error of uncertainty of measurement.

Errors of measurement

There is a limit to the accuracy of any measure, depending on the equipment used and the skill of the person making the measurement.

Some of the more common sources of errors in measurements are:

(a) Difficulty in reading scales and choice

(b) Parallax error

(c) Instrument design

(d) Instrument calibration

(e) Instrument zero-ing

(f) Deterioration

(g) Outside influences.

Absolute Error

If a value is measured several times, then the mean (average) can be calculated and an estimate of the accuracy can be given. The estimate of accuracy is called the absolute error.

Example

The length of an A4 page is measured a number of times and the result was found to be $297 \pm 0.5\text{mm}$. This indicates that your best estimate (the mean of your measurements) of the value is 297mm but it could lie somewhere between 296.5mm and 297.5mm. The absolute error is estimated at 0.5mm.

Fractional Error

$$\text{Fractional Error} = \frac{\text{Absolute Error}}{\text{Measured Value}} = \frac{\Delta l}{l}$$

So for the previous example of the A4 page, the fractional error would be:

$$Fractional\ Error = \frac{\Delta l}{l} = \frac{0.5}{297} = 0.0017$$

Therefore the length of an A4 (measured with a millimetre ruler) is 297 ± 0.0017mm.

Percentage Error

Fractional error is often converted into a percentage of the measured value.

$$\text{Percentage Error} = \text{Fractional Error} \times 100\%$$

So the length of our A4 page could also be expressed as 297 mm ± 0.17 %. (0.0017 × 100)

Exercise

Complete the following table. Give fractional error answers to 3 decimal places

Measurements taken and result calculated	Absolute Error	Fractional Error	% Error
(a) 7.3 ± 0.2g			
(b) 105 ± 5cm			
(c) 0.032 ± 0.003m			
(d) 6.2 ± 0.3kg			
(e) 0.050 ± 0.005mm			
(f) 73.5 ± 0.05s			

Answers

(a)	0.2g	0.027	2.7%
(b)	5cm	0.048	4.8%
(c)	0.003m	0.094	9.4%
(d)	0.3kg	0.048	4.8%
(e)	0.005mm	0.100	10%
(f)	0.05s	6.8×10^{-4}	0.07%

ERRORS IN CALCULATIONS

Error in a Calculated Value

When a result is calculated from a number of values, each of which has an uncertainty, then it follows that the calculated result is also subject to uncertainty.

Indeed, errors accumulate – that is, the **uncertainty increases**.

Error in a sum or difference

Suppose a measurement consists of two lengths added together.

$A = 4.0 \pm 0.2 \text{ cm}$ and $B = 2.0 \pm 0.1 \text{ cm}$

In adding *A* and *B*, the measured values are added, and also the uncertainties.

$$\therefore A + B = (4.0 + 2.0) \pm (0.2 + 0.1) \text{ cm}$$
$$A + B = 6.0 \pm 0.3 \text{ cm}$$

If the value that we seek is the difference between two lengths, for example *A* – *B*, then we calculate the difference between the measured values, **but we still add the uncertainties**.

$$\therefore A - B = (4.0 - 2.0) \pm (0.2 + 0.1) \text{ cm}$$
$$A - B = 2.0 \pm 0.3 \text{ cm}$$

The absolute error in a sum or difference is the sum of the absolute errors in each measurement.

Error in a product or quotient

There are many circumstances in which we need to multiply or divide measurements by other measurements.

For instance, surface area is usually a multiple of two measurements.

$$\text{Surface area}_{\text{rectangle}} = \text{length} \times \text{width}$$

When multiplying or dividing two measurements that have an uncertainty associated with them, we always **add** the fractional or percentages errors in both cases. Percentage errors have been used below.

$$\text{A} \times \text{B} = (\text{AB}) \pm (\text{AB} \times \%\ \text{error AB}) \quad \text{and} \quad \text{A} \div \text{B} = \left(\frac{\text{A}}{\text{B}}\right) \pm \left(\frac{\text{A}}{\text{B}} \times \%\ \text{error} \frac{\text{A}}{\text{B}}\right)$$

Example 1

Calculate the area of a rectangle and its associated absolute error from the following data:

Length $= 14.26 \pm 0.02\text{cm}$, Width $= 5.94 \pm 0.02\text{cm}$

Solution

$$\%\text{ error in area} = \%\text{ error length} + \%\text{ error width}$$

$$\%\text{ error length} = \frac{\text{absolute error length}}{\text{length}} \times 100 = \frac{0.02}{14.26} = 0.14\%$$

$$\%\text{ error width} = \frac{\text{absolute error width}}{\text{width}} \times 100 = \frac{0.02}{5.94} = 0.34\%$$

$$\%\text{ error for Area} = 0.14\% + 0.34\% = 0.48\% \text{ (always add the errors)}$$

$$\begin{aligned}\text{Absolute Error for Area} &= (\text{Area}) \pm (\text{Area} \times \%\text{ error of Area}) \\ &= (14.26 \times 5.94) \pm (14.26 \times 5.94) \times 0.48\% \\ &= 84.7 \pm 0.4\text{cm}^2\end{aligned}$$

Error in a power

If one quantity is raised to a power then the percentage error is added as many times as the power. eg, a value raised to a power of 3 would have its percentage error trebled. See next page for an example of a percentage error using powers.

Example 2

For a sphere of radius 5.0 ± 0.1 cm, calculate the volume of the sphere and its associated error.

Solution

$\text{Volume} = \frac{4}{3} \times \pi \times (\text{radius})^3$ Constants $\frac{4}{3} \times \pi$ are ignored for errors

% error in volume = 3 × % error in radius The radius is cubed, ∴ error is × 3

If radius was squared, error would be ×2

If radius was quadrupled, error would be ×4

$$= 3 \times \frac{\text{absolute error radius}}{\text{radius}} \times 100$$

$$= 3 \times \frac{0.1}{5} \times 100 = 6\%$$

$$\text{Volume} = \frac{4}{3}\pi r^3 = \frac{4}{3} \times \pi \times 5^3 = 524 \text{cm}^3$$

Absolute error in volume = volume × % error volume

= 524 × 6%

= 31 cm^3

∴ Volume = 524 ± 31 cm^3

Exercise

1. If A = 6.0 ± 0.1cm, and B = 3.4 ± 0.2cm, calculate:

(a) A + B, (b) A – B (c) A × B (d) A ÷ B

2. The mass of a square cube was found to be 60.7 ± 0.05 g. A side length of this cube = 1.5 ± 0.05 cm.

Calculate the density of the material and its associated error. (Density = Mass/Volume)

3. A cylinder has a height of 10.3 ± 0.05 cm. Its radius is 4.5 ± 0.05 cm.

Calculate the volume of the cylinder and its associated error? Volume $V = \pi r^2 h$

Answers

1. (a) 9.4 ± 0.3cm (b) 2.6 ± 0.3cm (c) 20.4 ± 1.54cm (d) 1.76 ± 0.13cm 2. 18 ± 2.6 g.cm^{-3}

3. 655 ± 18 cm^3

VECTORS: ADDITION

Most physics quantities can be classified into one of two groups. These groups are termed **vector** and **scalar** quantities.

Scalar quantities are completely defined by a magnitude and the relevant unit, eg 10 seconds.

Vector quantities, on the other hand, require magnitude and direction together with the relevant unit, eg. $10\ \text{kmh}^{-1}$ due North.

Some examples of vector and scalar quantities are shown below.

Vector	**Magnitude & direction**	***Scalar***	***Magnitude***
Velocity:	$90\ \text{ms}^{-1}$ South	Temperature:	7° C
Force:	15 N vertically down	Speed:	$90\ \text{kmh}^{-1}$
Displacement:	100 m to the right	Money:	$44
Acceleration:	$6\ \text{ms}^{-2}$ East	Distance:	100 km
		Current:	70 amp

Representation of vector quantities

Vectors are depicted diagrammatically by an arrow, whose length varies with the magnitude of the vector (drawn to scale!) and which points in the appropriate direction. A negative vector is simply a positive vector pointing the other way. In the diagram below, East is assumed to be positive.



The beginning and end of a vector line may be identified by letters such as **AB** or **XY** (see above). In such cases the vector may be referred to as $\overrightarrow{AB}$ or $\overrightarrow{XY}$. The little arrow above the letters identifies that the symbols represent a vector, but **not** the direction of the vector.

Magnitude

It is sometimes required to use only the **magnitude** (size) of a particular vector. In such cases, to distinguish that it is only the magnitude that is being used, we use the **modulus** symbol $|\ |$

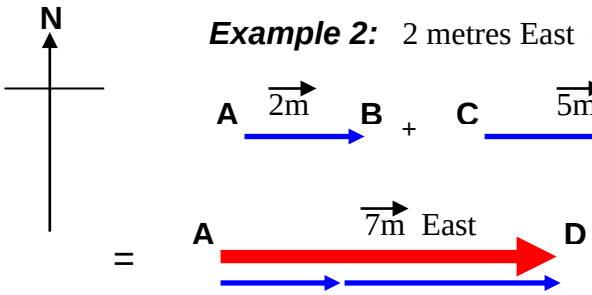
Example 1

The magnitude of 30ms^{-1} due South is $\left|\overrightarrow{30ms^{-1}S}\right| = 30\text{ms}^{-1}$

The magnitude of 18 Newtons West is $\left|\overrightarrow{18\,N\,West}\right|$ = 18 Newtons

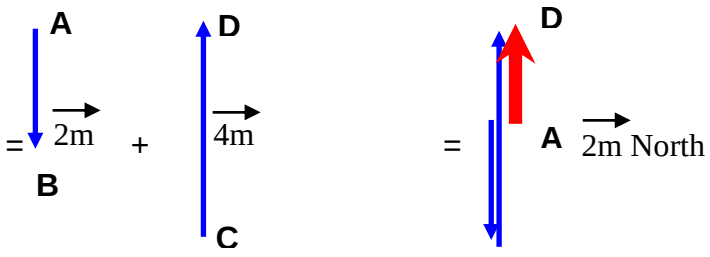
Adding vectors

The addition of two vectors is performed by placing the **tail** of the second vector at the **head** (the arrow end) of the first vector. The **vector sum** or **resultant vector** (shown in **red**) is represented in magnitude and direction by the beginning of vector **AB** to the end of the vector **CD**, i.e. vector **AD**. See examples on the following page.

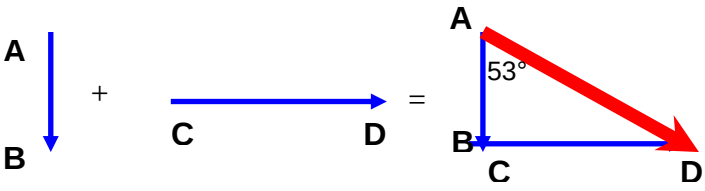


Example 2: 2 metres East + 5 metres East

Example 3: 2 metres South + 4 metres North



Example 4: 3m/s South + 4 m/s East



= 5 m/s South 53° East (127° T)
(use Pythagoras and trigonometry)

Exercise

1. (a) To 60 km East add 25 km East. (b) To 60 km East add 25 km West.
2. (a) To 30 km West add 40 km North, (b) To 30 km West add 30 km South.
3. A ship sails 40km North East, and then changes direction and sails 40km South East. Find the displacement of the ship (vector sum).
4. Two forces, 100 Newtons North and 100 Newtons East, are acting away from the same point and are at 90º to each other. Find their vector sum.
5. A plane is flying due North at 300 km h^{-1} and a westerly wind of 25 km h^{-1} is blowing it off course. Calculate the true speed and direction of the plane.

Answers

1. (a) 85 km East, (b) 35 km East (a) 50 km North 37º West (323ºT) (b) 42 km South West (225ºT)
3. 56 km East 4. 141 Newtons North East. 5. 301 kmh^{-1} North 5º East (005ºT).

SUBTRACTION OF VECTORS

Subtracting vectors

When subtracting or finding the change in a vector, the initial value is taken away from the final value. The change in velocity Δv, for example, is the final velocity v_f minus the initial velocity v_i, or:

$$\vec{\Delta v} = \vec{v_f} - \vec{v_i}$$

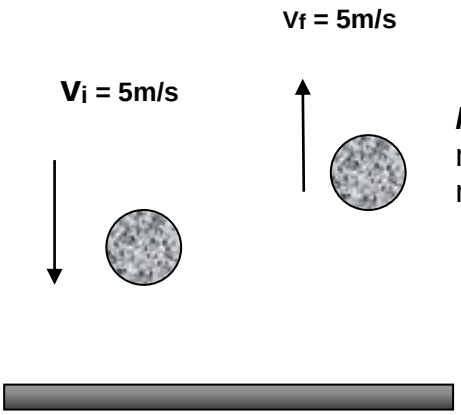
Other symbols used: initial velocity u
final velocity v

Putting it another way we have

$$\vec{\Delta v} = \vec{v_f} - \vec{v_i} = \vec{v_f} + (-\vec{v_i})$$

Thus, the change in velocity is the same as the final velocity plus the negative of the initial velocity. ***Recall:*** The negative of a vector is simply a vector that points in the opposite direction to the original vector.

Example 1 A golf ball is dropped onto a concrete floor and strikes the floor at 5 m/s. It then rebounds at 5 m/s.



What is the change in **speed** of the ball?

$$\Delta v = \text{final speed} - \text{initial speed} = 5 - 5 = 0$$

Note: Since speed is a scalar quantity, the direction of motion of the ball is not a consideration, only its magnitude. Hence the change in speed is zero.

What is the change in **velocity** of the ball?

$$\Delta v = v_f - v_i$$

$$\Delta v = 5 \uparrow - 5 \downarrow$$

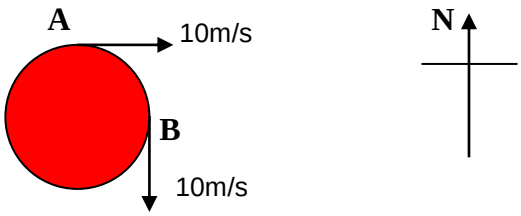
$$= 5 \uparrow + -5 \uparrow = 10 \uparrow$$

Since velocity is a vector, direction **is** a consideration. Hence vector subtraction is carried out.

Answer Change in velocity is 10 m/s up.

Example 2

A car is travelling at a uniform speed of 10m/s in a circle as shown below. Find the (a) change in **speed,** and (b) change in **velocity** of the car as it turns from A to B.



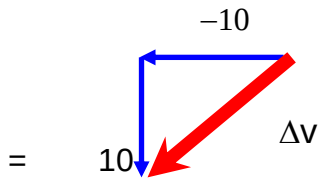
(a) Since speed is a scalar (direction is irrelevant), then the change in speed is given by:

$$(\text{final speed})_B - (\text{initial speed})_A = 10 - 10 = 0$$

(b) Change in velocity is given by

$$\Delta v = v_B - v_A$$

$$= 10\downarrow - 10\rightarrow = 10\downarrow + (-10)\leftarrow$$



$\Delta v = \overrightarrow{14.2}$ m/s South West (or 14.2 m/s 225 T)

Multiplying vectors

When a vector is multiplied by a scalar (number) the magnitude of the vector changes accordingly. For example, if *"x"* is a displacement of 5 m North, then $4x$ is 20 m North.

Exercise

1. Carry out the following vector subtractions:
   (a) 24 m N minus 18 m North, (b) 48 m N minus 22 m South, (c) 13.8 ms^{-1} East minus 9.4 ms^{-1} North.

2. A ball is thrown in an Easterly direction and strikes a wall at 12.5 ms^{-1}. If it rebounds at the same speed, what is its (a) change in speed, (b) change in velocity.

3. From $\overrightarrow{14.14}$ kmh^{-1} East subtract $\overrightarrow{14.14}$ kmh^{-1} North.

4. What is the change in velocity when an aeroplane travelling at 250 kmh^{-1} West alters its course to 350 kmh^{-1} North?

Answers

Exercise 1. (a) 6m North (b) 70 m North (c) 16.7 ms^{-1} S56E (or 124ºT) 2. (a) zero (b) 25 ms^{-1} West

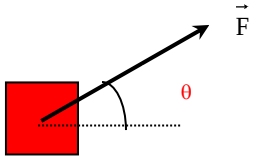
3. 20 kmh^{-1} South East 4. 430 kmh^{-1} North 35º East (035ºT).

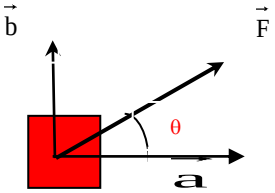
VECTORS: RESOLUTION

Vector Components

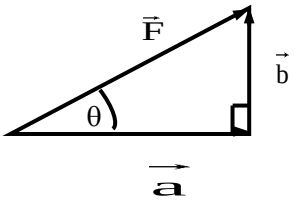
Force is a vector. As with other types of vectors (displacement, velocity), a force vector can be thought of as being the sum of several others. Imagining that a single force is made up of several forces can greatly simplify calculations, particularly using two imaginary ones at right angles to each other.

For instance, if you pull on an object with a force $\vec{F}$ (see diagram below left), the effect is the same as two other people pulling with forces $\vec{a}$ and $\vec{b}$ at right angles to each other (see diagram below right).





The vector triangle showing the force $\vec{F}$ and its component vectors $\vec{a}$ and $\vec{b}$ is shown below.



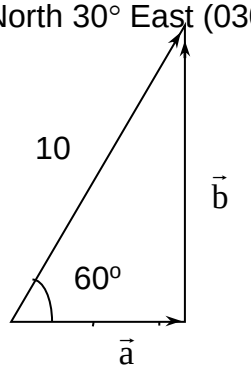
The force $\vec{F}$ can be imagined as having two parts, or **components,** $\vec{a}$ and $\vec{b}$ that are at right angles to each other. $\vec{a}$ is usually called the horizontal component and $\vec{b}$ the vertical component of $\vec{F}$. The components are often called **rectangular components** because they are at right angles to

For the diagram above, it can be seen that

$\sin\theta = \frac{b}{F}$ so $b = F\sin\theta$ and $\cos\theta = \frac{a}{F}$ so $a = F\cos\theta$

Example 1:

Calculate the rectangular components of a force of 10 N that acts in a direction of North 30° East (030ºT).



The force and its rectangular components are shown. One component, $\vec{a}$, is acting to the east, the other, $\vec{b}$, to the north.

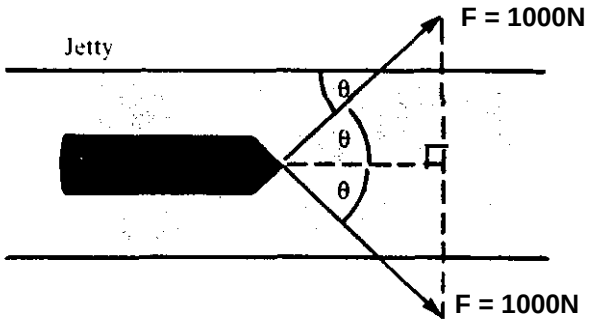
$\vec{a} = \vec{F}\cos\theta$
$= 10\cos 60°$
$= 5\text{N}$

$\vec{b} = \vec{F}\sin\theta$
$= 10\sin 60°$
$= 8.7\text{N}$

Thus, the 10N force can be resolved into two rectangular components: 5 N to the East and 8.7 N to the North.

Example 2:

A small boat is being towed at constant speed along a canal by two men walking along the banks on opposite sides pulling with equal forces of 1000N at equal angles on ropes attached to the boat. θ = 60º. Calculate the force each man pulls on the rope (a) parallel to the bank, and (b) at right angles to the bank.



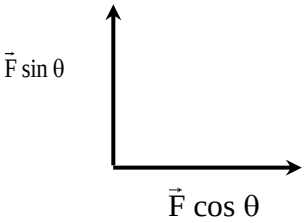
The 1000N force in each rope has two components:

(a) a component force **parallel** to the bank of

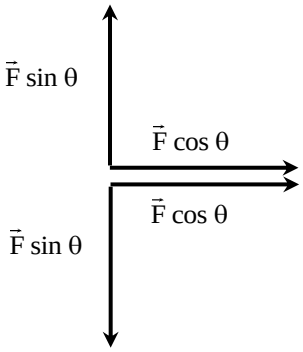
$\vec{a} = \vec{F}\cos\theta = 1000 \times \cos 60^\circ = 500\text{N}$

(b) a component force at **right angles** to the bank of

$\vec{b} = \vec{F}\sin\theta = 1000 \times \cos 30^\circ = 866\text{N}$



Note that the two components at right angles to the bank are equal in size, but are in opposite directions, so they cancel each other. So the boat will only move parallel to the banks.



The two components parallel to the banks are in the same direction so the force pulling the boat forwards is

$2 \times \vec{F}\cos\theta = 2 \times 500 = 1000\text{N}$.

The balancing force causing the boat to move at constant speed is the friction or drag of the boat through the water acting in the opposite direction.

Exercise

1. What is the horizontal component of a force of 11N acting at 60º to the horizontal?
2. What is the northerly component of a wind blowing at 15ms^{-1} from the South East?
3. What is the vector whose components are 3N horizontally and 4N vertically?
4. What are the vertical and horizontal components of a force of 25N acting at 30 º to the horizontal?
5. A plane taking off leaves the runway at 32º to the horizontal, travelling at 180kmh^{-1}. How long will it take to climb to an altitude of 1000m?

Answers

1. 5.5N 2. 11ms^{-1} 3. 5N 50º above the horizontal. 4. 12N vertical, 22N horizontal. 5. 37.8s

LINEAR MOTION: GRAPHS

Linear motion refers to the motion of an object in a straight line.

Displacement (*x*)
When an object is moved from one point to another it is said to be displaced. If we more to Dandenong we can say we have been displaced 32 km from Melbourne, but to completely define our position we must say we are 32 km South East of Melbourne, ie we have given both a magnitude (32 km) and a direction (South East). Displacement is a vector quantity.
Note: Displacement is sometimes referred to as 'position'. See exercises at the end of this worksheet.

Distance (*d*)
Distance is the magnitude (size) of displacement, but has no direction so it is a **scalar** quantity, ie my distance is 60 km, but my displacement is 60 km due West.

Velocity (*v*)
Velocity is the rate of change of displacement per second, ie

$v = \frac{x}{t}$ where x is in metres, and t is in seconds. Velocity has units of ms^{-1}.

Since velocity involves displacement it is also a vector quantity.

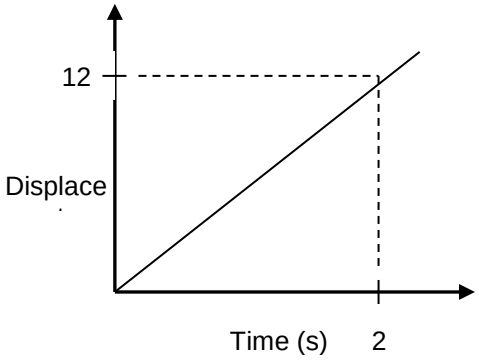
Speed (*v*)
Speed is the magnitude (size) of velocity, but has no direction so it is a scalar quantity, ie

My speed is 60 kmh^{-1}, but my velocity is 60 kmh^{-1} due East.
The unit of speed is also ms^{-1}.

Acceleration (*a*)
Acceleration is the rate of change of velocity per second, ie.

$a = \frac{v}{t}$ where v is in ms^{-1} and t is in seconds. So acceleration has units of ms^{-2}.

Graphing Motion: Displacement-Time graphs



A person runs 12 metres in 2 seconds.

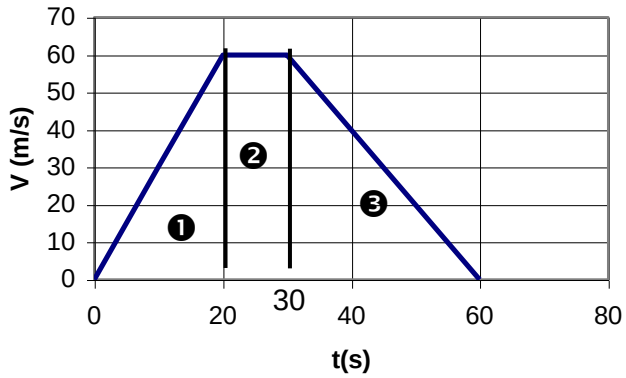
$$v = \frac{x}{t} = \frac{12}{2} = 6ms^{-1}$$

Note: the gradient of a Displacement-Time graph gives the velocity

Gradient of distance-time graph = speed Gradient of displacement-time graph = velocity

Velocity-Time graphs

The velocity-time graph below shows a car which accelerates uniformly from rest to 60 ms^{-1} in 20 seconds, then travels at a constant velocity of 60 ms^{-1} for the next 10 seconds, then decelerates uniformly to rest in 30 seconds. The total journey took 60 seconds.



Area under a velocity-time graph

Area = length × width = metres/second × seconds = metres.

Area under a velocity-time graph gives the displacement. Area under a speed-time graph gives the distance.

We can divide the graph into 3 distinct sections and calculate the area for each.

Area ❶ Triangle = ½ × base × height = ½×20×60 = 600 metres.

Area ❷ Rectangle = Length × Width = 60 × 10 = 600 metres.

Area ❸ Triangle = ½ × base × height = ½×30×60 = 900 metres.

Total displacement = Total area = 600 + 600 + 900 = 2100 metres

Gradient of a velocity-time graph

$$\text{Gradient} = \frac{\text{Rise}}{\text{Run}} = \frac{\text{Change in Velocity}}{\text{Time}} = \text{Acceleration}$$

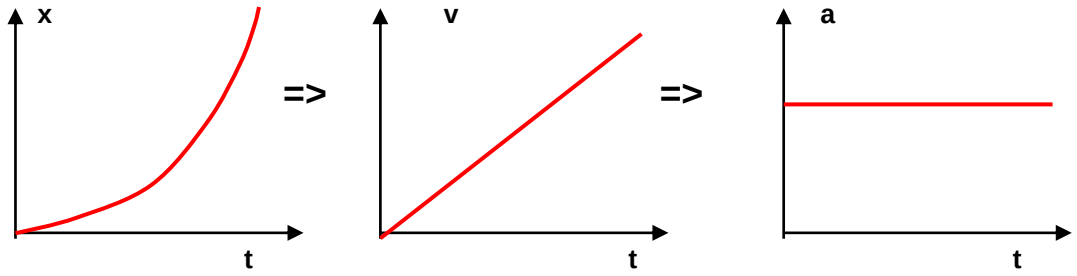
0-20 seconds: Gradient = 60/20 = 3 ms^{-1}. Car is accelerating at 3 ms^{-1}.

20-30 seconds: Gradient = 0/10 = 0 ms^{-1}.
Note: No slope ∴ No acceleration. Car is travelling at a constant velocity of 60m/s.
30-60 seconds. Gradient = –60/30 = –2 ms^{-1}

Note: Negative slope ∴Negative acceleration, or deceleration. Car is slowing down.

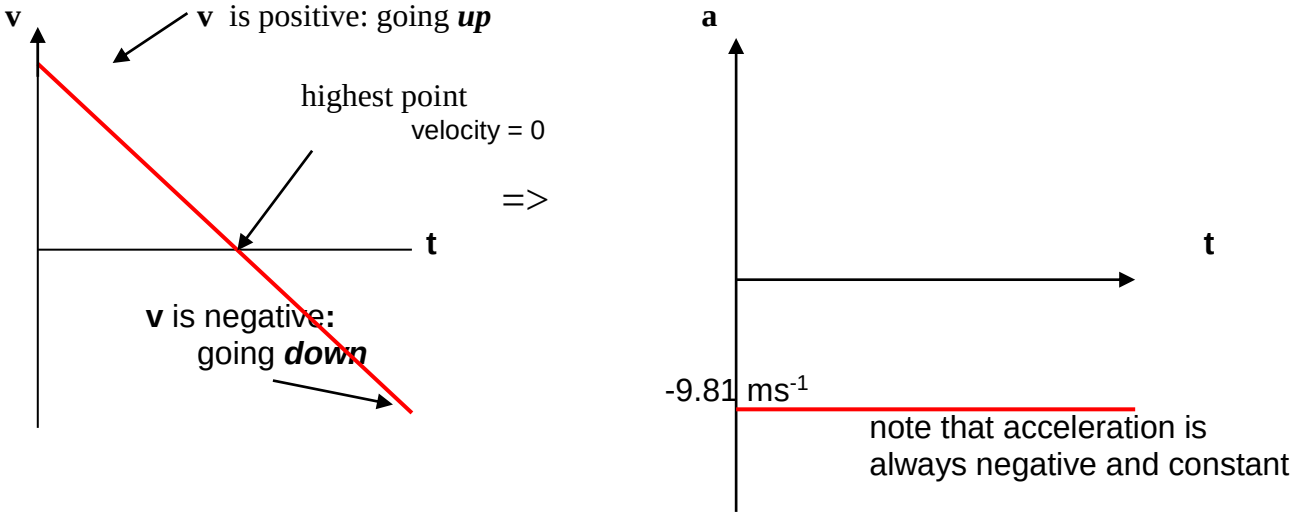
Some further examples

Example 1: A car accelerates from a stationary position. The acceleration is constant (uniform).



Note that the slope of the "x-t" graph gives a "v-t" graph, and the slope of the "v-t" graph gives an "a-t" graph.

Example 2: A ball is thrown vertically upwards.

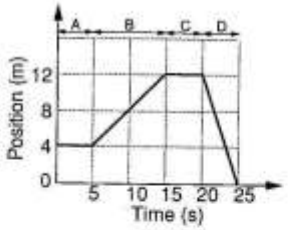


Summary

Graph type Found from	*x – t*	*v - t*	*a - t*
Gradient	Velocity	Acceleration	Meaningless
Area under graph	Meaningless	Displacement	Change in velocity

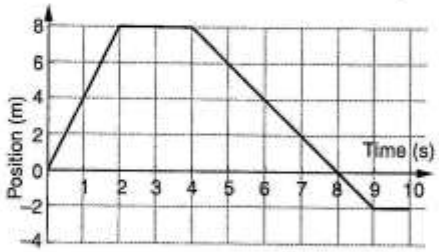
Exercise

1 The graph shows the position of a dancer moving across a stage.

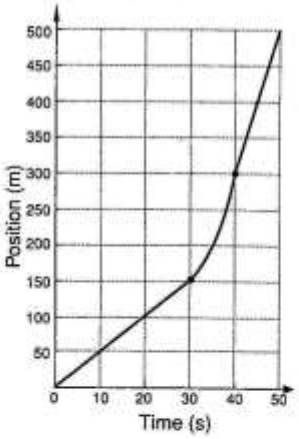


a What was the starting position of the dancer?
b In which of the sections (A–D) is the dancer at rest?
c In which of the sections is the dancer moving in a positive direction?
d In which of the sections is the dancer moving with a negative velocity?

The following information relates to questions 2–6. The graph represents the straight line motion of a radio-controlled toy car.

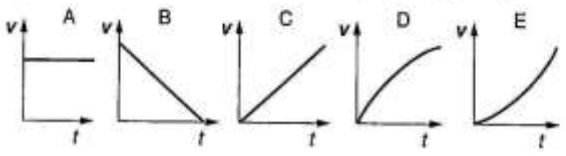


2 Describe the motion of the car.
3 What was the position of the car after:
a 2 s? **b** 4 s? **c** 6 s? **d** 10 s?
4 When did the car return to its starting point?
5 What was the velocity of the car:
a during the first 2 s? **b** after 3 s?
c from 4 s to 8 s? **d** at 8 s?
e from 8 s to 9 s?
6 During its 10 s motion, what was the car's:
a distance travelled? **b** displacement?
7 The following position–time graph is for a cyclist travelling along a straight road.

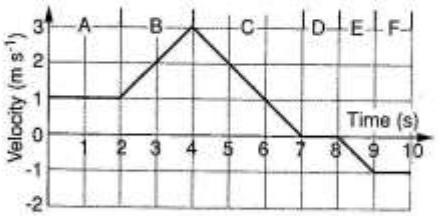


a Describe the motion of the cyclist.
b What was the velocity of the cyclist during the first 30 s?
c What was the cyclist's velocity during the final 10 s?
d Calculate the cyclist's instantaneous velocity at 35 s.
e What was the average velocity of the cyclist between 30 s and 40 s?

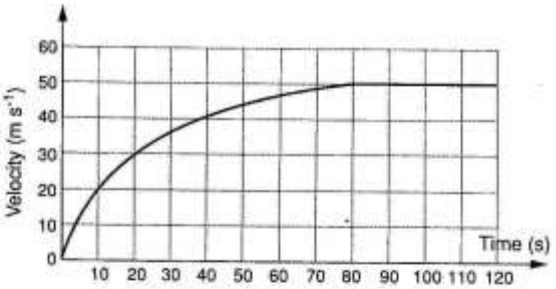
8 Which of the velocity–time graphs A–E best represents the motion of:
a a car coming to a stop at a traffic light?
b a swimmer moving with constant speed?
c a cyclist accelerating from rest with constant acceleration?
d a car accelerating from rest and changing through its gears?



The following information relates to questions 9–11. The following graph shows the motion of a dog running along a footpath. In this problem, north is considered to be positive.

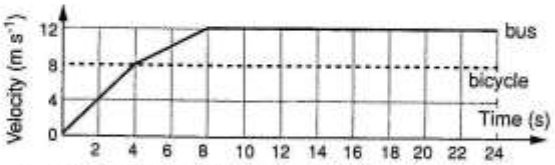


9 Describe the motion of the dog during these sections of the graph.
a A **b** B **c** C **d** D **e** E **f** F
10 Calculate the displacement of the dog after:
a 2 s **b** 7 s **c** 10 s.
11 Plot a position–time graph of the dog's motion.
12 The straight-line motion of a high speed intercity train is shown below.



a How long does it take the train to reach its cruising speed?
b What is the acceleration of the train 10 s after starting?
c What is the acceleration of the train 40 s after starting?
d What is the displacement of the train after 120 s?

13 The velocity–time graphs for a bus and a bicycle travelling along the same straight stretch of road are shown below. The bus is initially at rest and starts moving as the bicycle passes it.

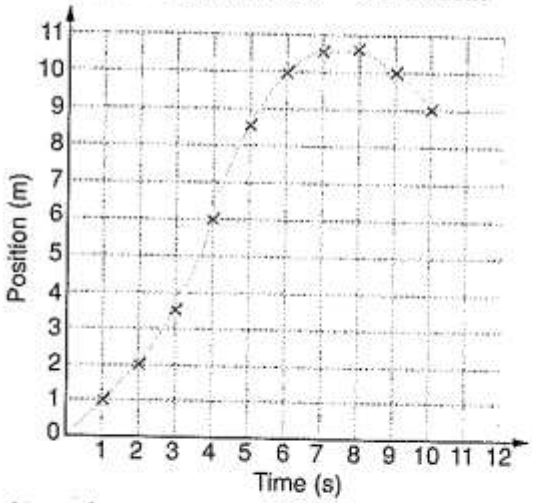


a Calculate the initial acceleration of the bus.
b When does the bus first start gaining ground on the bicycle?
c At what time does the bus overtake the bicycle?
d How far has the bicycle travelled before the bus catches it?
e What is the average velocity of the bus during the first 8 s?

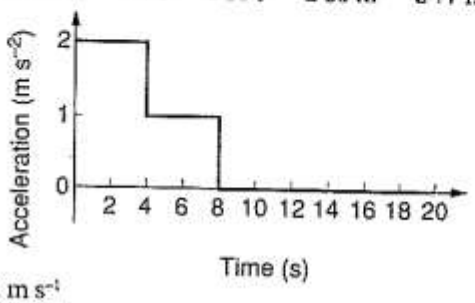
14 **a** Draw an acceleration–time graph for the bus discussed in question 13.
b Use your acceleration–time graph to determine the change in velocity of the bus over the first 8 s.

Answers

1 a +4 m **b** A, C **c** B **d** D
2 The car initially moves in a positive direction and travels 8 m in 2 s. It then stops for 2 s. The car then reverses direction for 5 s, passing back through its starting point after 8 s. It travels a further 2 m in a negative direction before stopping after 9 s.
3 a +8 m **b** +8 m **c** +4 m **d** −2 m **4** 8 s
5 a +4 m s^{-1} **b** 0 **c** −2 m s^{-1} **d** −2 m s^{-1} **e** −2 m s^{-1}
6 a 18 m **b** −2 m
7 a The cyclist travels with a constant velocity in a positive direction for the first 30 s, travelling 150 m during this time. Then the cyclist speeds up for 10 s, travelling a further 150 m. Finally the cyclist maintains this increased speed for the final 10 s, covering another 200 m in this time.
b +5 m s^{-1} **c** +20 m s^{-1} **d** approx. 13 m s^{-1} **e** +15 m s^{-1}
8 a B **b** A **c** C **d** D
9 a Running north at 1 m s^{-1}
b Increasing speed from 1 m s^{-1} to 3 m s^{-1} while running north
c Running north but slowing to a stop
d Stationary
e Accelerating from rest to 1 m s^{-1} while running south
f Running south at 1 m s^{-1}
10 a 2 m north **b** 10.5 m north **c** 9 m north
11



12 a 80 s **b** −1.3 m s^{-2} **c** −0.5 m s^{-2} **d** −4900 m
13 a +2 m s^{-2} **b** 4 s **c** 10 s **d** 80 m **e** +7 m s^{-1}
14 a



b +12 m s^{-1}

LINEAR MOTION: EQUATIONS OF MOTION

Equations with uniform acceleration: horizontal motion

We have already used the formula $v = \frac{s}{t}$ to describe motion where the velocity is constant.

This formula cannot be used in situations where there is uniform accelerated motion.

Recall from graphs of motion that the uniform acceleration 'a' of an object is given by the gradient of a velocity-time graph:

$$acceleration\ \ a = \frac{Rise}{Run} = \frac{change\ in\ velocity}{time\ taken} = \frac{\Delta v}{t} = \frac{v_f - v_i}{t} \quad \therefore\ at = v_f - v_i$$

Since $a \times t = v_f - v_i$

$\boxed{v_f = v_i + at}$ Similar mathematical gymnastics can be used to show that:

$\boxed{x = v_i t + \tfrac{1}{2} a t^2 \qquad x = v_f t - \tfrac{1}{2} a t^2 \qquad v_f^{\,2} = v_i^{\,2} + 2ax \qquad x = \tfrac{1}{2}(v_i + v_f) t}$

where x = displacement(m), v_i = initial velocity(ms^{-1}), v_f = final velocity (ms^{-1}), t = time taken (s)

Other symbols may be used to denote x *(s)*, 'v_i' (u , v_o) and v_f (v).

Note: It is often necessary to specify right or left as the positive or negative direction when doing these problems since we are dealing with vector quantities.

Example

A truck has an acceleration of $0.8ms^{-2}$. Find (a) the time taken and (b) the distance travelled for the truck to reach a velocity of $20ms^{-1}$ from a starting velocity of $3ms^{-1}$.

(a) List the data first: $v_i = 3$, $v_f = 20$, $a = 0.8$, $t = ?$

Now find the appropriate equation that includes this data: $v_f = v_i + at$

From which $t = \frac{v_f - v_i}{a} = \frac{20-3}{0.8} = 21.3s$

(b) List the data first: $v_i = 3$, $v_f = 20$, $a = 0.8$, $x = ?$

Now find the appropriate equation that includes this data: $v_f^{\,2} = v_i^2 + 2ax$

From which $x = \frac{v_f^{\,2} - v_i^{\,2}}{2a} = \frac{20^2 - 3^2}{2 \times 0.8} = 244.4m$

Vertical motion under gravity

The acceleration of a falling object near the Earth's surface is 9.8ms^{-2}. For instance, a coin that is dropped from rest will have a velocity of 9.8ms^{-1} after 1 second, 19.6ms^{-1} after 2 seconds, and so on. The quantity, 'g', can also be used when describing gravitational acceleration. For example, an astronaut will experience an acceleration of 39.2ms^{-2}, or 4g, at take-off.

Since the acceleration of a freely falling object is uniform (constant), we can use the same equations as we did with horizontal motion.

Note: It is often necessary to specify up or down as the positive or negative direction when doing these problems as we are dealing with vector quantities.

Example 1

A construction worker accidentally knocks a brick from a building so that it falls vertically a distance of 50m to the ground. Calculate (a) the time taken for the brick to reach the ground, (b) the speed of the brick as it hits the ground.

(a) List the data first: $v_i = 0$ (at rest), $x = 50$, $a = 9.8$, $t = ?$

Now find the appropriate equation that includes this data: $x = v_i t + \frac{1}{2} a t^2$

$$x = v_i t + 0.5 \times 9.8 \times t^2 \Rightarrow 50 = 0 + 0.5 \times 9.8 \times t^2 \Rightarrow t = 3.2s$$

(b) List the data first: $v_i = 0$ (at rest), $x = 50$, $a = 9.8$, $t = 3.2$, $v_f = ?$

Now find the appropriate equation that includes this data: $v_f = v_i + at$

$$v_f = v_i + at = 0 + 9.8 \times 3.2 \Rightarrow v_f = 31 ms^{-1}$$

Example 2

A ball is thrown up into the air with a velocity of 30ms^{-1}. Find (a) the maximum height reached by the ball, (b) the time taken for the ball to reach its maximum height.

Take **up** as positive, and **down** as negative.

(a) List the data first: $v_i = +30$ (velocity is **up**), $v_f = 0$ (ball is stationary at top of flight), $a = -9.8$ (acceleration is always **down**), $x = ?$

Now find the appropriate equation that includes this data: $v_f^2 = v_i^2 + 2ax$

$$0 = 30^2 + 2 \times -9.8 \times x \Rightarrow x = 46m$$

(b) List the data first: $v_i = +30$ (velocity is **up**), $v_f = 0$ (ball is stationary at top of flight), $a = -9.8$ (acceleration is always **down**), $x = 45$, $t = ?$
Now find the appropriate equation that includes this data: $v_f = v_i + at$
$0 = 30 + -9.8 \times t \Rightarrow t = 3.1s$

Note: by symmetry, the ball will also take 3.1 seconds to return to the ground, assuming that air resistance is ignored. Total time of flight = 6.2 seconds.

Exercise **Use g = 9.8ms^{-2}**

1. A car, starting from rest, moves with an acceleration of 2ms^{-2}. Find (a) the velocity at the end of 20 seconds, and (b) the distance covered in that time.

2. With what uniform acceleration does a spacecraft, starting from rest, cover 1000 metres in 10 seconds?

3. A cyclist, starting from rest, moves with an acceleration of 3ms^{-2}. (a) In what time will it reach a velocity of 30m/s, and (b) what distance does the cyclist cover in this time?

4. An object starts with a velocity of 100m/s and decelerates (slows down) at 2ms^{-2}.
   (a) When will its velocity be zero, and (b) how far will it have gone?

5. A book is knocked off a bench and falls vertically to the floor. If the book takes 1.0s to fall to the floor, calculate:
   a. its speed as it lands.
   b. the height from which it fell.
   c. the distance it falls during the first 0.5s.
   d. the distance it falls during the final 0.5s.

6. A champagne cork travels vertically into the air. It takes 4.0s to return to its starting position.
   a. How long does the cork take to reach its maximum height?
   b. What was the maximum height reached by the cork?
   c. How fast was the cork travelling initially?
   d. What the speed of the cork as it returned to its starting point?
   e. Describe the acceleration of the cork at each of these times after its launch:
      (i) 1.0s (ii) 2.0s (iii) 3.0s.

Answers

1. (a) 40 ms^{-1} (b) 400m 2. 20 ms^{-2} 3. (a) 10s (b) 150m 4. (a) 50s (b) 2500m

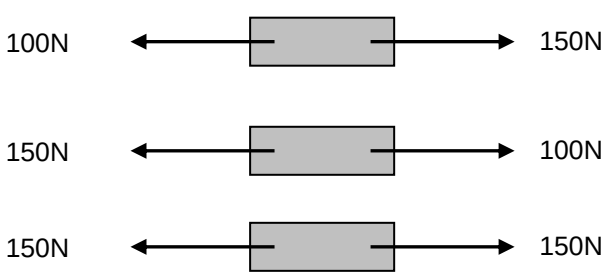
5. (a) 9.8ms^{-1} (b) 4.9m (c) 1.2m (d) 3.7m 6. (a) 2.0s (b) 19.6m (c) 19.6ms^{-1} (d) 19.6ms^{-1}

(e) (i) 9.8ms^{-2} down (ii) 9.8ms^{-2} down (iii) 9.8ms^{-2} down

NEWTON'S LAWS OF MOTION

Net (or resultant) force ΣF

Examples



Σ F = 150 – 100 = 50N to the Right

Σ F = 150 – 100 = 50N to the Left
∴ unbalanced force

Σ F = 150 – 150 = 0 Stationary or moving at constant speed (see below)

Note: Force is a vector quantity, so direction must be taken into account.

Newtons 1st law of motion

Every object continues to be at rest, or continues with constant velocity, unless it experiences an unbalanced force.

Seatbelts are fitted in cars to take into account Newton's 1st Law of Motion. If, for instance, you had to brake suddenly and you were not wearing your seatbelt, then by Newton's 1st Law, you would continue to travel at a uniform speed – the speed of the car just before braking – until you made contact with the windscreen. That is, the sum of the forces acting on you was zero at the instant you braked even though the car was being acted upon by a braking force.

Newtons 2nd law of motion

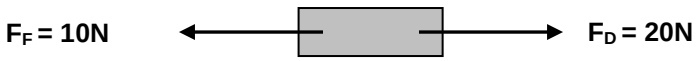
This law relates to the sum (ΣF) of the forces acting on an object and the acceleration produced as a consequence of this resultant force. It is given by:

$$\Sigma F = ma$$

where ΣF is the sum of the forces (in Newtons), 'm' is the mass of the object (in kg), and 'a' is the acceleration (in ms^{-2}). Note: ΣF must have the same direction as "a".

When doing Newton's 2nd law problems draw all forces acting **on** the object in question.

Example Toy car of mass 2 kg has a driving force F_D of 20N. The frictional force F_F acting on the car is 10N. Find the car's acceleration



$\Sigma F = ma$ where $\Sigma F = F_D - F_F = 20 - 10 = 10N$

Hence $\Sigma F = ma \Rightarrow 10 = 2 \times a \Rightarrow a = 5ms^{-2}$

Newtons 3rd law of motion

When two objects interact, the forces they exert on each other are equal in size, but opposite in direction. These pairs of forces are called action/reaction forces.

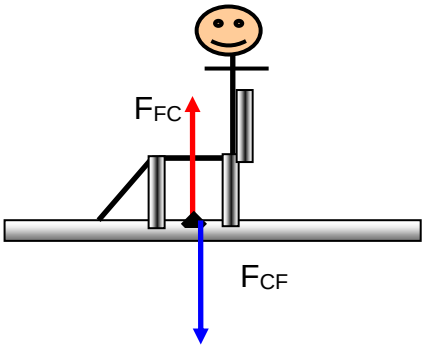
You need to appreciate that each force in the action-reaction pair acts on different objects. This is different to Newton's 2nd law where you analyse forces that are acting on the same object. Note that each force is equal in size but opposite in direction to its paired force.

Did you know: You can only walk forward because the ground pushes you forward as a result of you pushing backwards against the ground!! This is an example of Newton's Third Law.

Normal force F_N

A normal force acts at right angles to the surface with which it is in contact. For instance, if you are currently sitting on a chair reading this handout, then the chair is in contact with the floor and so there is a contact force of the chair acting on the floor given by F_{CF}. By Newton's 3rd law the floor will exert an equal but opposite force on the chair of F_{FC}. F_{FC} is also called the normal force F_N. See diagram below.

$F_{FC} = -F_{CF}$ where the negative sign indicates that these forces are acting in opposite directions. Recall your notes on vectors.

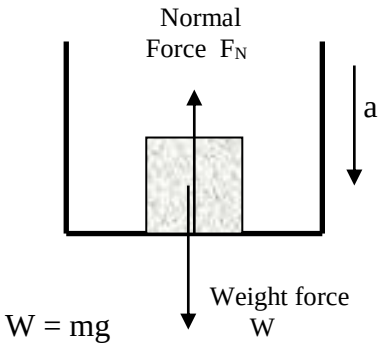


$F_{FC} = F_N$

Applications of Newton's laws.

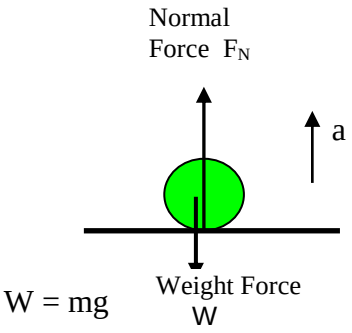
A starting point when analysing Newton's 2nd law problems is to draw all forces acting on the object in question.

Objects in Lifts



- If accelerating **down** (as shown at left): $\Sigma F = mg - F_N = ma$
- If accelerating **up**: $\Sigma F = F_N - mg = ma$
- If not accelerating at all: $\Sigma F = F_N - mg = 0 \Rightarrow F_N = mg$

Objects bouncing



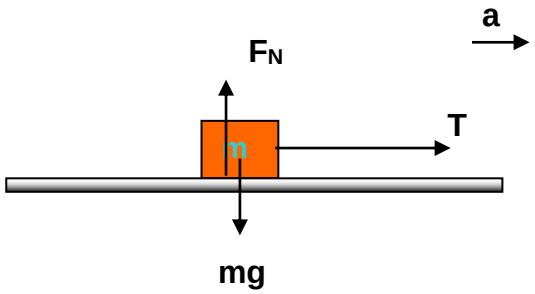
During the bounce (the instant the ball is in contact with the surface):

- $F_N > mg$, because $a > 0$ and is **upwards**. Therefore there is a resultant force acting which is upwards.
- F_N can often be much larger than mg
- If object is **stationary** $\Sigma F = F_N - mg = 0 \Rightarrow F_N = mg$

Object being pulled along a surface

T = tension in rope a = acceleration of mass

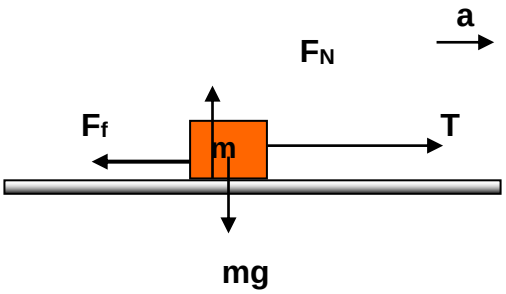
Smooth surface (no friction)



Horizontally: $\Sigma F = T = ma$

Vertically: $\Sigma F = F_N - mg = 0$
$\Rightarrow F_N = mg$

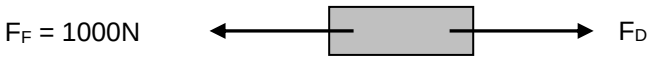
Rough surface (friction)



Horizontally: $\Sigma F = T - F_f = ma$

Vertically: $\Sigma F = F_N - mg = 0$
$\Rightarrow F_N = mg$

Example A car of mass 800kg accelerates from rest to 20ms^{-1} in 8.0s. The resistance forces acting on the car total 1000N. Find (a) the acceleration of the car, and (b) the driving force of the car.



(a) Acceleration is given by the change in velocity divided by the time taken, or

$a = (v - u)/t = (20 - 0)/ 8 = 2.5\ ms^{-2}$ Recall: $v = u + at$

(b) Net Force $\Sigma F = F_D - F_F = ma$ or $F_D - 1000 = 800 \times 2.5 \Rightarrow F_D = 3000N$

The driving force of the car is 3000N

Exercise

1 What resultant force is needed to give a mass of 6.4 kg an acceleration of 2.4ms^{-2} West?

2 If a resultant force of 48N produces an acceleration of 1.2 ms^{-2} on an object, what is the mass of the object?

3 A resultant force of 5.0N acts on an object and causes it to reach a velocity of 4.0 ms^{-1} in 2.5s. What is the mass of the object?

4 An object of mass 6.0kg is at rest on a rough horizontal table. A horizontal force of 2.4N acts on the mass to the East. The mass reaches a speed of 1.2 ms^{-1} in a distance of 2.0m.

(a) Calculate the acceleration of the mass.
(b) What is the net horizontal force acting on the object?
(c) What is the frictional force acting on the object?

Answers

1. 15N West 2. 40kg 3. 3.1kg 4 (a) 0.36ms^{-2} East (b) 2.2N East (c) 0.24N West.

STATICS

Static Equilibrium

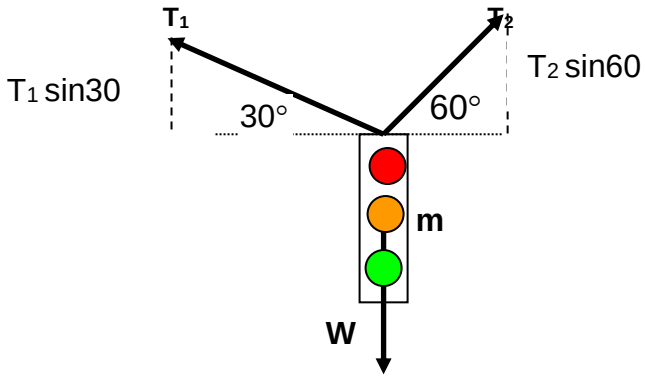
Structures that are stable are said to be in static equilibrium. There are two types of static equilibrium: translational equilibrium and rotational equilibrium.

Translational equilibrium means that the sum of the forces (or net force) acting on the system is zero, ie, Forces Up = Forces Down, and Forces Right = Forces Left, or

$$\Sigma F = 0$$

Example 1

Consider a set of traffic lights of mass m suspended above an intersection by a pair of wires of negligible mass, as shown below.



Since this structure is in equilibrium the magnitude of the components of forces upwards must equal the magnitude of the components of forces downwards, or

$\Sigma F = 0$

Resolving T_1 and T_2 vertically in the diagram above:

$T_1 \sin 30 + T_2 \sin 60 - W = 0$, or $T_1 \sin 30 + T_2 \sin 60 = W = mg$ Note : Up is +

Alternatively, we could resolve T_1 and T_2 horizontally.
Since there is no horizontal motion:

$\Sigma F = 0$

Resolving T_1 and T_2 horizontally in the diagram above:

$T_2 \cos 60 - T_1 \cos 30 = 0$, or $T_2 \cos 60 = T_1 \cos 30$ Note : RIGHT is +

Note that the vertical weight force W (= mg) when resolved horizontally equals zero ($W \times \cos 90 = 0$).

Rotational equilibrium: Moment (or Torque) of a Force

The turning effect or moment M of a force F acting at a distance r from the axis of rotation of a rigid body is given by

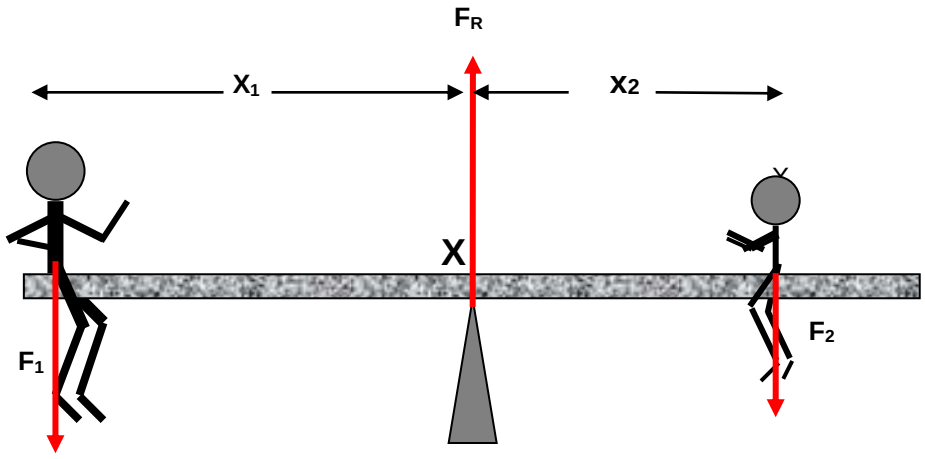
$$M = F \times r$$

where F and r are mutually perpendicular (at right angles) to each other.

For rotational equilibrium to occur the sum of the moments acting on the structure is zero, or

$$\Sigma M = 0$$

Example 2 Consider the seesaw example below.



For translational equilibrium: $\Sigma F = 0$ or

$F_R - F_1 - F_2 = 0$ where UP is +. Alternatively,

$F_R = F_1 - F_2$

For rotational equilibrium: $\Sigma M = 0$ and taking moments about X in the diagram above:

$+(F_2 \times x) - (F_1 \times x_1) = 0$ where clockwise moments are +

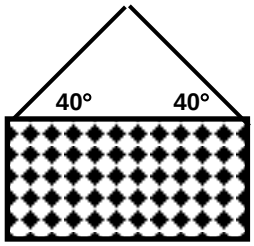
An example of a system that is not in rotational equilibrium is a **moving** see-saw. If you are sitting on one end and an elephant (!) decides to sit on the other end, then it will rotate!!

Exercise

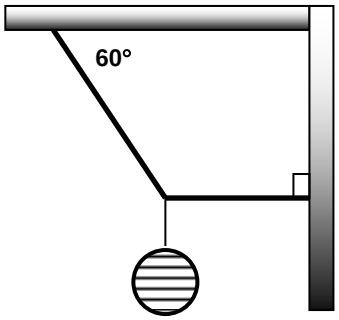
Take g = 9.8ms^{-2}

1. In Example 1, T_1 = 1000N, and T_2 = 2000N. Using vertical forces only calculate:
   (a) the maximum **weight** of the traffic lights that can be supported by the cables.
   (b) the maximum **mass** of the traffic lights that can be supported by the cables.

2. A picture is hung as shown in the following diagram. If the hanging wire has a breaking strength of 40 N, what is the maximum possible mass of the picture?



3. A 100 gram electric light is supported by two cables, one at an angle of 60° with the ceiling, and the other being perpendicular to the wall. Assuming the mass of each cable is negligible, calculate the tension in each cable.



4. Two children are balanced on a see-saw which is supported in the middle. One child weighs 200 N and is 1.2 m from the axis, while the other child is seated 1.5 m from the axis. How much does the second child weigh? Ignore the mass of the see-saw.

5. Two children wish to make a see-saw from a 5.0 m plank of wood. The children weigh 25 kg and 20 kg. They each wish to sit right on the ends of the plank. Where should the plank be supported in order for it to balance?

Answers

1. (a) $T_1 = 2.2\times10^3$N, (b) 224kg 2. T = 5.2kg 3. T1 = 1.13 N, T2 = 0.57 N.
4. W = 160 N 5. 2.2 m from the heavier child.

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FRICTION

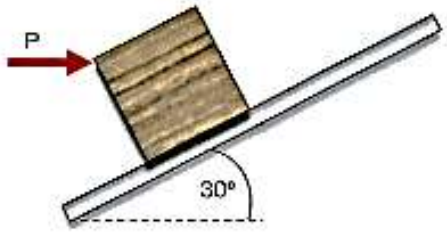
Question

The crate shown below has a mass of 50kg and the coefficient of static friction between the crate and the plane is $\mu_s = 0.25$. $g = 9.81 ms^{-2}$

(a) Calculate the minimum force P required to stop the crate from sliding down the plane.

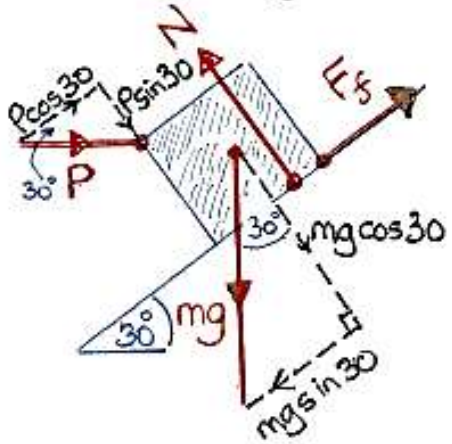
(b) Calculate the minimum force P required to push the crate up the plane.

(Hibbeler, R.C, 2010, *Engineering Mechanics: Statics*, Pearson)



Worked Solution (a)

Free Body Diagram



NOTE:

- Normal force N must act a distance from the crate's centre to counteract the tipping effect caused by P
- Friction force F_f must act up the plane to prevent crate from slipping down.
- $F_f = \mu_s N$

Resolving forces parallel to plane $\nearrow$ $\Sigma F_{\parallel} = 0$

$P\cos 30 + F_f - mg\sin 30 = 0$

$P\cos 30 + \mu_s N - 50(9.81)\sin 30 = 0$

$0.87P + 0.25N - 245.25 = 0 \Rightarrow N = \dfrac{245.25 - 0.87P}{0.25}$

Resolving forces perpendicular to plane $\nwarrow$ $\Sigma F_{\perp} = 0$

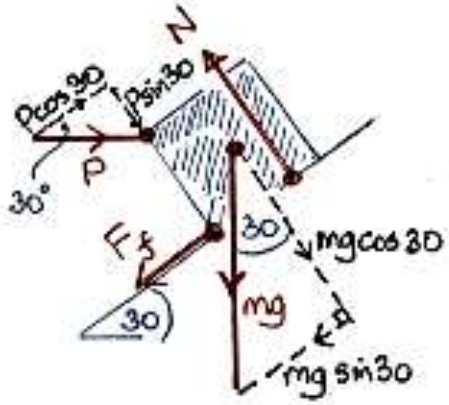
$N - P\sin 30 - mg\cos 30 = 0$, and substituting for N above

$\dfrac{245.25 - 0.87P}{0.25} - 0.5P - 424.78 = 0$

$\Rightarrow P = 140\,N$

Worked Solution (b)

Free Body Diagram



NOTE :
Friction force F_f must act down the plane to resist crate being pushed up the plane.

Resolving forces parallel to plane $\nearrow$ $\Sigma F_{\parallel} = 0$

$P\cos 30 - F_f - mg\sin 30 = 0$

$P\cos 30 - \mu_s N - 50(9.81)\sin 30 = 0$

$0.87P - 0.25N - 245.25 = 0 \Rightarrow N = \dfrac{0.87P - 245.25}{0.25}$

Resolving forces perpendicular to plane $\nwarrow$ $\Sigma F_{\perp} = 0$

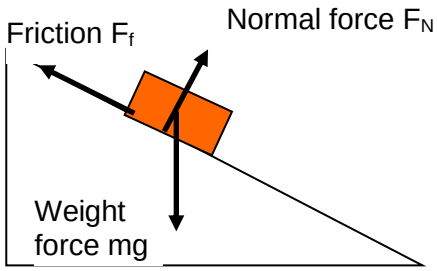
$N - P\sin 30 - mg\cos 30 = 0$, and substituting for N above

$\dfrac{0.87P - 245.25}{0.25} - 0.5P - 424.78 = 0 \Rightarrow P = 474\,N$

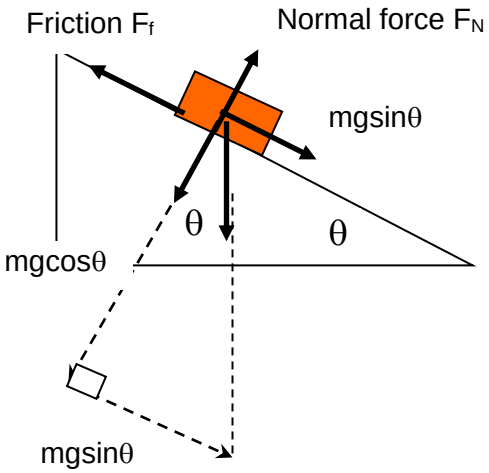
FORCES ON SLOPES

The normal force of an object placed on a sloping surface is always perpendicular to the surface. Another vital feature is that just as we can analyse the horizontal and vertical components of the motion of an object separately, we can look at components parallel to and perpendicular to the surface of a sloping surface as well. See diagrams below.

Forces acting on a block on an inclined plane



Weight force resolved into its components

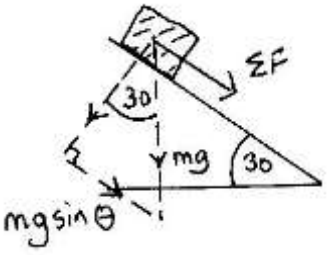


Note that with the above diagrams:

- Weight = mg; acts through the centre of mass.
- Normal force F_N is always at right angles to the surface.
- Friction acts to oppose sliding motion (eg, if the mass were being dragged uphill, friction would act downhill)
- The weight force is resolved into 2 components:
  - mg cosθ perpendicular to plane, and
  - mg sinθ parallel to the plane.
- The resultant force ΣF down the slope is given by $\Sigma F = mg \sin\theta - F_f$ where F_f is friction
- The resultant force ΣF perpendicular to the slope is zero, hence: $mg\cos\theta = F_N$

Example

A toy car of mass 50g travels down a smooth incline at 30 degrees to the horizontal. Friction may be ignored. Calculate (a) the net force acting on the car as it rolls down the slope, and (b) the force of the incline on the car as it travels down the slope.

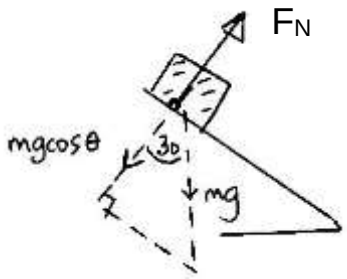


(a) $\Sigma F = ma = mg \sin\theta$
where $mg \sin\theta$ is the component of the force **parallel** to the slope.

Note: the surface is frictionless (smooth), therefore the only force allowing the car to roll down the incline is the component of the gravitational force 'mg sin θ' .

$\Sigma F = m \times g \times \sin\theta = 50 \times 10^{-3} \times 9.8 \times \sin 30 = 0.25N$

Note: grams have been converted into kilograms



(b) The force of the incline on the car is a force that acts **perpendicular** to the slope, ie, the normal force F_N

This is equal to mg cos θ

$F_N = mg \cos\theta = 50 \times 10^{-3} \times 9.8 \times \cos 30 = 0.43N$

Exercise

Take g = 9.8ms^{-2}

1 A boy and his skateboard of total mass 60kg coasts down a **frictionless** ramp at an angle of 30º to the horizontal.
(a) Calculate the normal force acting on the boy and skateboard.
(b) Calculate the force acting on the boy and skateboard parallel to the ramp.

2 The boy now coasts down another ramp, but this time the ramp has a **rough** surface.
(a) Calculate the normal force acting on the boy and skateboard.
(b) Calculate the force acting on the boy and skateboard parallel to the ramp.
(c) If the ramp has a frictional force of 54N, what is the net force acting on the boy and the skateboard?
(d) Calculate the acceleration of the boy.
(e) If the boy started from rest, and the ramp is 4m long, what was the speed of the boy at the bottom of the ramp?

Answers 1(a) 509N (b) 294N 2. (a) 509N (b) 294N (c) 240N (d) 4.0ms^{-2} (e) 5.6 ms^{-2}

Friction

Static Friction F_f Friction that acts on an object and stops it from moving.

$F_f = \mu_s N$ where μ_s is the coefficient of static friction, N is the normal reaction

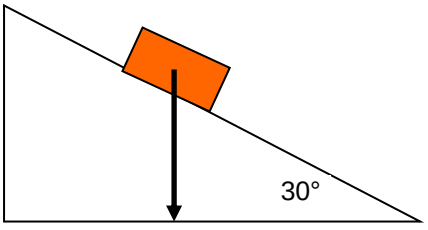
Kinetic Friction F_k Friction that acts on an object during motion.

$F_k = \mu_k N$ where μ_k is the coefficient of kinetic friction, N is the normal reaction.

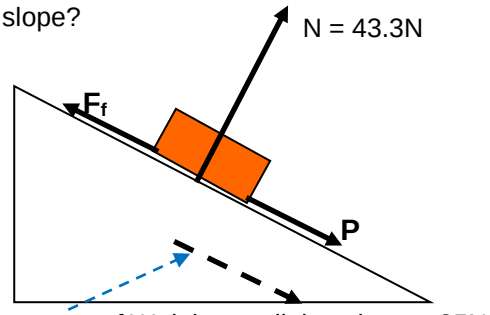
Example

An object of weight 50N is stationary on a slope which has a coefficient of static friction of 0.8, and a coefficient of kinetic friction of 0.7. See diagram below left.

(a) What force P is required to *just* move it down the slope?



Weight = 50N



Component of Weight parallel to slope = 25N

$F_f = \mu_s N = 0.8 \times 43.3 = 34.6N$

Resolving forces parallel to slope, see diagram above right:

ΣF = 0: P + 25 – F_f = 0, P + 25 – 34.6 = 0, P = 9.6N

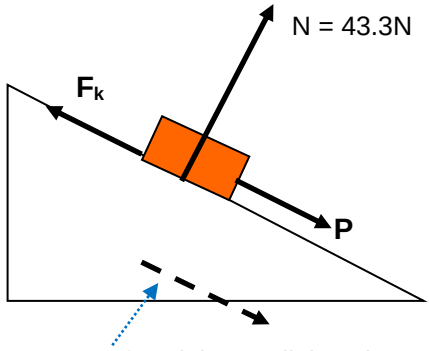
(b) What force P is required to keep it moving down the slope at a constant velocity?

$F_k = \mu_k N = 0.7 \times 43.3 = 30.3N$

Resolving forces parallel to slope:
Constant velocity, hence $\Sigma F = 0$:

$P + 25 - F_k = 0, \quad P + 25 - 30.3 = 0,$

$P = 5.3N$



Exercise

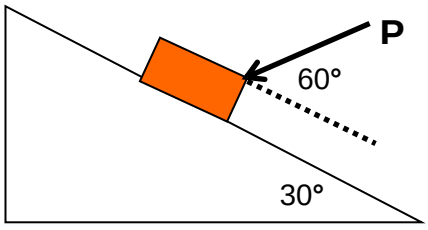
Take g = 9.81ms^{-2}

1 An object of mass 50kg is sliding down a slope which is at an angle of 30° to the horizontal. Find the objects's acceleration if (a) there is no friction (b) the coefficient of kinetic friction is 0.15

2 A 10 kg block is stationary at the top of a ramp 21 m long, inclined at 10° to the horizontal. The coefficient of static friction is 0.9. (a) What force is required to *just* move the block down the ramp? (b) If the force is increased to 100N after it has travelled 1m down the ramp and the coefficient of kinetic friction is 0.8, how long will it take the block to slide down the rest of the ramp?

Read the 'Friction' worksheet before attempting question 3.

3 The block shown below has a mass of 20kg and the coefficient of static friction between the block and the ramp is 0.5. Calculate, to one decimal place, (a) the minimum force P required to stop the crate from sliding down the ramp, (b) the minimum force P required to push the block up the ramp.



Answers

1. (a) 4.9ms^{-2} (b) 3.6ms^{-2} 2. (a) 69.9N (b) 4.5s 3 (a) 26.2N (b) 366.2N

MOMENTUM

The momentum p of an object is the product of its mass m and velocity v, or

$$p = m \times v$$

The unit of momentum is $kgms^{-1}$

Momentum is a vector quantity, so it is important to remember that direction must be taken into account when doing problems on momentum.

Change in Momentum

Consider an object of mass 'm' changing its velocity from v_i to v_f in time Δt under the action of a resultant force ΣF.

From Newton's second law of motion $\Sigma F = ma$,

$$\Sigma F = \frac{mv_f - mv_i}{\Delta t} \quad \text{since} \quad a = \frac{v_f - v_i}{\Delta t}, \quad \text{or}$$

$$\Sigma F \times \Delta t = mv_f - mv_i$$

In other words, when a resultant force ΣF acts on an object for a time duration of Δt there is a change in momentum given by $(mv_f - mv_i)$, where mv_i is the initial momentum and mv_f is the final momentum. This change in momentum is written as Δp, or

$$\Delta p = p_f - p_i = mv_f - mv_i$$

where p_i is the initial momentum and p_f is the final momentum.

Note: one must be careful when calculating Δp since change in momentum involves subtracting one vector from another.

Impulse

As shown above, when a resultant force ΣF acts on an object for a time Δt the object experiences a change in momentum. The product of this resultant force and time is called the impulse I of the force.

$$I = \Sigma F \times \Delta t = \Delta p = mv_f - mv_i$$

The unit of impulse is Ns.

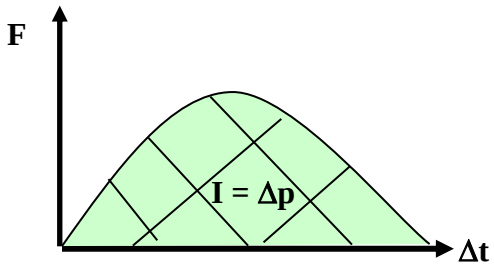
It can be shown that since change in momentum is equal to impulse, then kgms^{-1} is equivalent to Ns.

Note:

1. As momentum and impulse are vectors, a sign convention in problems on momentum and impulse is essential.
2. A negative sign for the change in momentum indicates a loss of momentum; a positive sign indicates a gain in momentum.
3. Make sure you can do vector subtraction calculations.
4. Remember that ΣF is the resultant force.

Impulse is given by the area under an F-Δt graph

The area under a force-time graph gives the impulse of a force. This is very useful when the force is non-uniform (see below).



I = FΔt is central to much of modern car design. For instance, if a test car travelling at 60km/h crashes into a wall or a large balloon (!) the change in momentum Δp for both will be the same (why?).

Since $F\Delta t = m\Delta v$ then $F\Delta t = k$ (constant)

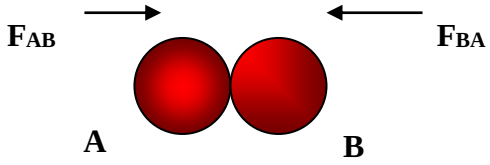
Hence $F \propto \frac{1}{\Delta t}$

For the wall crash: Δt is small, therefore F is large, ie, a larger force is acting for a smaller time..

For the balloon crash: Δt is large therefore F is small, ie, a smaller force is acting for a longer time.

Conservation of Momentum

When two balls A and B collide, the action of A on B is equal and opposite to that of B on A. (Newtons 3rd Law)



Hence the rate of change of momentum of A is equal and opposite to the rate of change of momentum of B. Since the time of contact is the same for both, then the change in momentum of A is equal and opposite to the change in momentum of B.

That is, the total momentum before impact equals the total momentum after impact, or .

$$\Sigma p = 0$$

This is known as the law of conservation of momentum. The total momentum is the same before, during and after impact, or

$$m_A u_A + m_B u_B = m_A v_A + m_B v_B$$

where m_A = mass of A, m_B = mass of B, u_A = velocity of A before impact, u_B = velocity of B before impact, v_A = velocity of A after impact, v_B = velocity of B after impact.

Note:

- Always draw a diagram when doing momentum problems.
- A sign convention (+, –) is essential (momentum and impulse are vectors). Be careful with negative and positive signs!
- If, for instance, the two cars collide and stay together after the collision, then the momentum of the two cars before the collision is equal to the momentum of the locked-together cars after the collision.
- Mathematically, problems on collisions or explosions are similar, except that for an explosion, the momentum of the system before the blast is often zero.
- In closed systems no external forces act. For example, a spacecraft colliding in deep space experiences no external force because there is no gravitational field. Systems close to the Earth's surface are systems that are not closed because the external force acting is the Earth's gravitational force. To "make" them closed one has to include the Earth as part of the system.

Momentum transfer involving the Earth

The following two situations describe how momentum is transferred to the Earth.

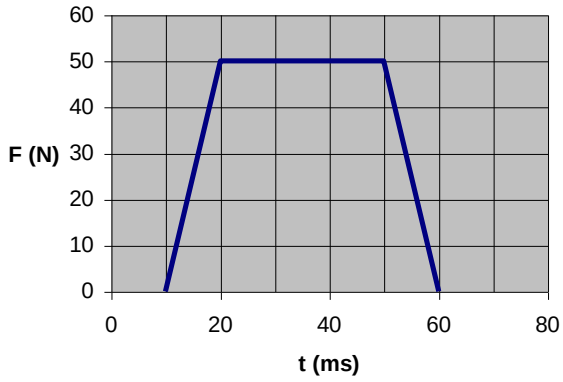
- A ball is thrown up: it rises against gravity and then slows down and stops, losing momentum due to the earth. The ball starts to fall under the influence of gravity, speeding up thereby giving the earth an equal and opposite momentum change. The falling ball hits the ground. The momentum is not lost. It is transferred to the earth.
- A bicycle on the flat is slowed down due to friction. The loss of momentum of the bike is transferred to the earth which gains an equal and opposite momentum.

Exercise

1 A tennis ball of mass 100g hits the wall horizontally at 8.0ms^{-1} east and rebounds at 6.0ms^{-1}. The contact time with the wall is 0.07s.
Calculate (a) the impulse on the ball by the wall, (b) the change in momentum of the ball. State the magnitude and direction for (a) and (b)

2 Referring to Question1 above, calculate (a) the force exerted on the ball by the wall, and (b) the force exerted on the wall by the ball. State the magnitude and direction for (a) and (b)

3 The graph shown below shows how force varies with time for a miniature crash test dummy of mass 2 kg moving to the right. It is involved in a collision with a large concrete block set into the ground which brings it to rest. Calculate the dummy's initial speed.



4 A truck of mass 2500kg travelling at 20ms^{-1} west collides head on with a car of mass 800kg travelling in the opposite direction at 15 ms^{-1}. The two vehicles become locked together.
(a) What is the total momentum of the two vehicles before the collision? Assume the truck's motion (west) is positive.
(b) What is the speed and direction of the car and truck immediately after the collision?

5 A racing car negotiating a tight bend at 30 kmh^{-1} collides with a crash barrier. The air bag in his car inflates and the time taken for it to inflate is 0.16s. The driver's head has a mass of 7.0kg.
Explain why the driver is less likely to suffer head injury in a collision with the air bag than if his head collided with the car dashboard, or other hard surface.

Answers

1 (a) 1.4 Ns West (b) 1.4 kgm/s West
2 (a) 20N West (b) 20N East
3 10m/s 4. (a) 3.8×10^4 kgm/s (b) 12 m/s West
5
- $F\Delta t = k$ (constant), since $m\Delta v$ is constant

  Hence, $F \alpha \frac{1}{\Delta t}$
- In other words, the force of impact is inversely proportional to the time of impact
- For the dashboard impact: Δt is small, therefore F is large, resulting in serious injury
- For the air bag impact: Δt is large, therefore F is small, resulting in a much less serious injury

WORK, ENERGY AND POWER

Energy exists in many different forms, eg, kinetic energy E_k, potential energy U_g, electrical energy, and elastic (or spring) energy E_s. A fundamental principle of nature is that energy cannot be created or destroyed, only transformed or transferred.

Kinetic Energy KE

Kinetic energy KE, is energy associated with motion.

$$KE = \tfrac{1}{2} mv^2$$

where m = mass in kilograms (kg), and v is the speed (m/s). The unit of kinetic energy is the joule (J)

If an object's speed is doubled, then its kinetic energy is quadrupled. This is because the kinetic energy of an object is proportional to its (speed)^2. Similarly, tripling a car's speed will increase its kinetic energy ninefold! Naturally this has very dangerous implications for cars travelling at high speeds.

Gravitational Potential Energy PE

Gravitational potential energy PE is energy that is "stored" due to its position in a gravitational field.

$$PE = mgh$$

where m = mass in kilograms (kg), g = gravitational acceleration (m/s^2),
and h = height above a reference point (m).

Work W and Energy E

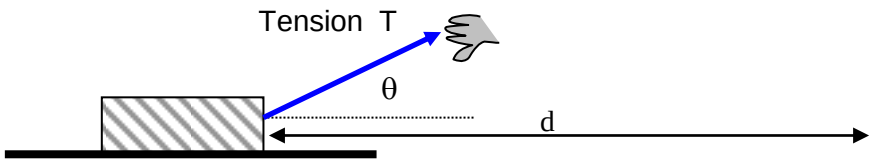
A body that has energy may transfer some, or all, of its energy to another body. The total amount of energy remains constant (conserved) even if it has been transformed to another type. The amount of energy transformed (ΔE) is called work W. The body losing energy does work, the body gaining energy has work done on it.

Work is given by the force multiplied by the displacement through which the force acts, or:

$$\text{Work} = \text{Change in Energy} = \text{Force} \times \text{displacement}$$
$$W = \Delta E = F \times d$$

where F = force (N), d = displacement (m).
Note: Work is a scalar quantity. The unit of work is the Joule (J)

Consider a force that acts at an angle. For instance, when a block is being pulled with tension T by a string whose angle of inclination is θ to the horizontal.



If the block moves a displacement d metres, then:

$$W = \Delta E = F \times d = T \cos\theta \times d$$

where F = T cos θ, the component of T in the horizontal direction.
Observe how the tension has been resolved in the direction of travel, ie, in the horizontal direction T cos θ.

Note: if the force is perpendicular to the displacement no work is done.
The reason: $W = F \cos\theta \times d = F \cos 90 \times d = 0$, where θ (90°) is the angle between the force and the displacement through which the force acts. The implication of this is that if the force acts at an angle to the direction of motion of an object, then the force is less effective compared with if the same force was acting parallel to the object's motion.

If an object is acted on by a force, then the work done on it is equal to the change in kinetic energy of the object:

$$W = \Delta E = \tfrac{1}{2} mv_f^{\,2} - \tfrac{1}{2} mv_i^{\,2}$$

where "v_i" is the initial speed and "v_f" is the final speed

Force-Distance graphs

If the force is non-uniform then the formula $W = F \times d$ does not apply. Instead, we present the information as a force-distance graph and calculate the area beneath the graph to find the work done by the force.

Recall the other "area under a graph", which is the force-time graph that gives the impulse of a force. Again, these graphs are only useful if the force is not uniform.

Power P

Power P is the rate at which work is done

$$\text{Power} = \frac{\text{work done}}{\text{time taken}} = \frac{W}{t} = \frac{F \times d}{t} = F \times v$$

where v is the speed d/t. The unit of power is the Watt (W).

Note: the formula $P = F \times v$ can only be used if an object is travelling at a constant speed.

Conservation of Energy

A fundamental principle of nature is that energy is conserved, meaning that while it may change from one form to another the total amount is always the same. Similarly energy may be transferred from one type (eg, potential energy) to another (eg, kinetic energy) but again the overall amount stays constant.

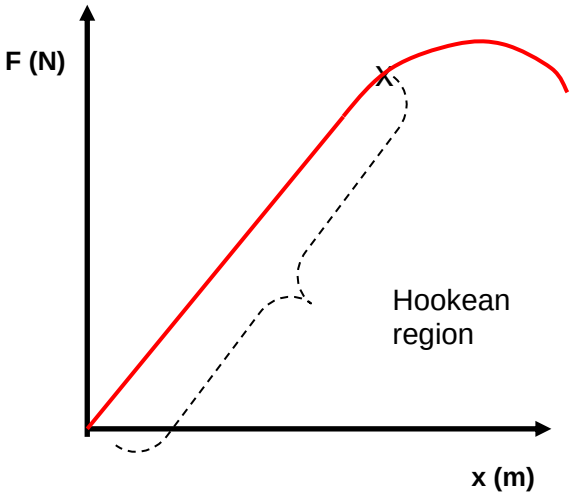
$$\Delta PE = \Delta KE$$

For instance, dropping a stationary ball of mass m from a height h involves a change in energy from potential to kinetic, or

$$\Delta(mgh) = \Delta(\tfrac{1}{2} mv^2)$$

Δ means "change in"

Hooke's Law and Elastic Potential Energy



Hooke's Law states that the force F applied to a spring or similar object is proportional to the spring's extension or compression.That is,

$$F = -k\,x$$

where k = force (or spring) constant (N/m)
x = extension or compression

The straight line (the "Hookean" region) passing through the origin shown in the graph at left is given by the slope of the F-x graph. It gives a measure of the stiffness of the material: the steeper the slope the stiffer the material.

Note: The negative sign in the formula above tells us that the "restoring force", the ability of a spring to pull back (or extend) to its original length, is in the opposite direction to the applied force.

Elastic limit

Objects may obey Hooke's Law initially (the straight line section of the graph), but will reach a point where they becomes non linear (no longer proportional). This point is called the elastic limit of the spring and is shown by the **X** on the graph above. The object may start to stiffen, it could weaken (like chewing gum), or it may eventually break. Stretching an object beyond the elastic limit produces permanent changes to the structure of the material such that even when the force is released it doesn't behave the same, and will not return to its original length.

Elastic (Strain) Potential Energy U_s

The elastic potential energy Us, also known as the strain energy, is the energy stored in an elastic material, and is given by:

$$U_s = \tfrac{1}{2}\,kx^2$$

where k = spring constant, x = extension (or compression) of the spring.

Force-Extension (or Compression) Graph

Just as the area under a force-displacement graph gives us the work done by a force (see above), so the area under a force-extension (or compression) graph allows us to calculate the work done by a spring.

Exercise

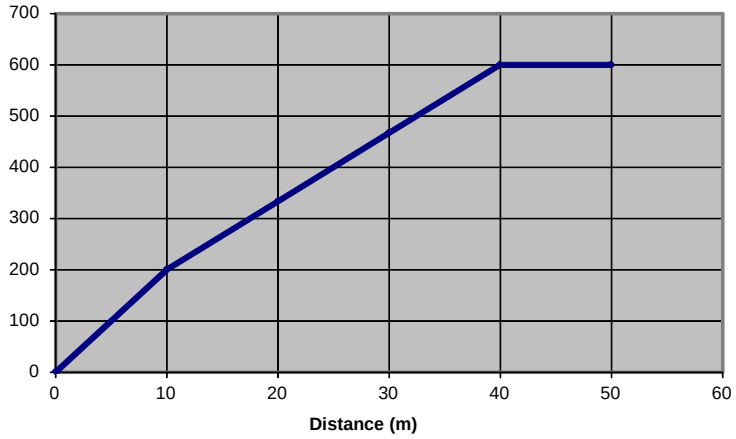
Take $g = 9.81 ms^{-2}$

1 (a) Calculate the work required for a man, of mass 80kg, to climb a 100m flight of stairs.
(b) How much power was developed by the man if he took 20 seconds to climb the stairs? Give your answers to two decimal places.

2 How much work is required to push a 50kg block along a floor, of coefficient of friction 0.6, at constant speed? Give your answer to one decimal place.

3 The graph below shows the pedalling force (in Newtons on the Y-axis) applied by a

cyclist at the start of a race against the distance travelled (in metres on the X-axis). The cyclist and bicycle have a combined mass of 80kg. Give your answers to two decimal places.

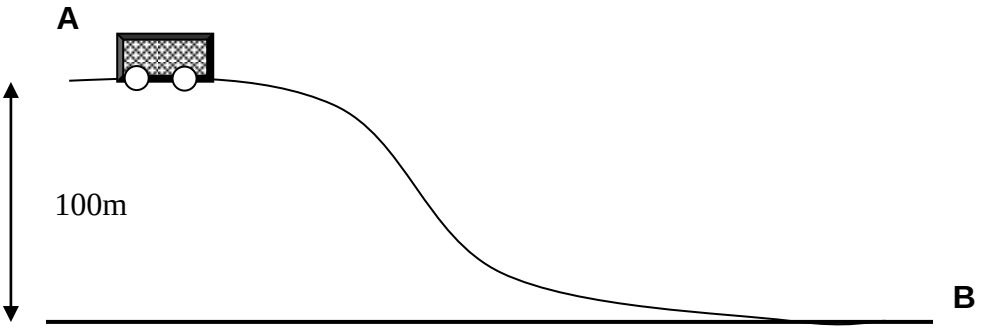


a. How much work does the cyclist do in the 50 seconds at the start of the race?

b. Calculate the power developed by the cyclist during the first 50m of the race if she took 10 seconds to cover this distance.

(c) Between 40 and 50 seconds, the cyclist hits a rough patch of track with a frictional force of 360N. What is her acceleration during this 10 seconds?

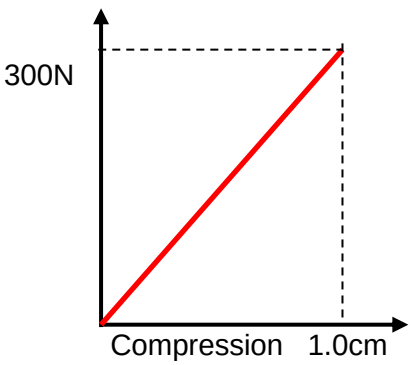
4. A car is shown at the top of an extremely steep hill, of height 100m, as shown below. The mass of the car and driver is 1200kg. The car is at rest at point A.



(a) Assuming frictional forces are ignored, calculate the speed that the car has at point B if it is allowed to roll down the hill without the driver applying the brakes.

(b) Assuming the car applies the brakes and does 1.0×10^6 J of work against friction as it rolls down the hill, what is the speed of the car at B now?

5. The graph below shows how the force applied by a pinball spring plunger changes as it is compressed during a pinball game.



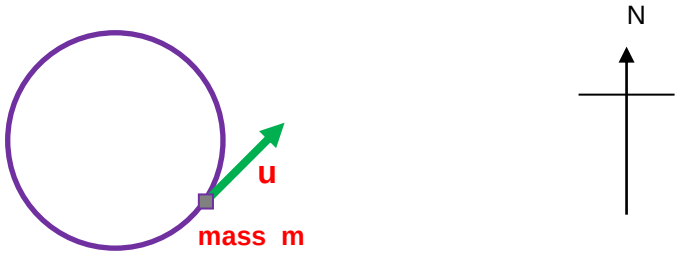
The plunger is compressed 1cm and then released. If the pinball has a mass of 50 grams:
(a) how much energy is stored in the spring prior to release?
(b) what is its speed at the instant it leaves the plunger?
Give your answers to one decimal place.

Answers (a)1 (a) 9.61×10^5J (b) 4.81×10^4W 2. 294.3J 3. (a) 1.82×10^4J (b) 1.82×10^3 W
3 (c) $3ms^{-2}$ 4. (a) 45 m/s (b) 18 m/s 5 (a) 1.5J (b) 7.7m/s

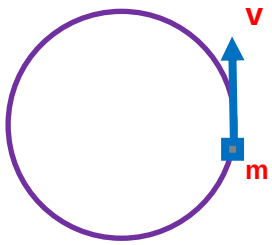
CIRCULAR MOTION

Radial (Centripetal) Acceleration

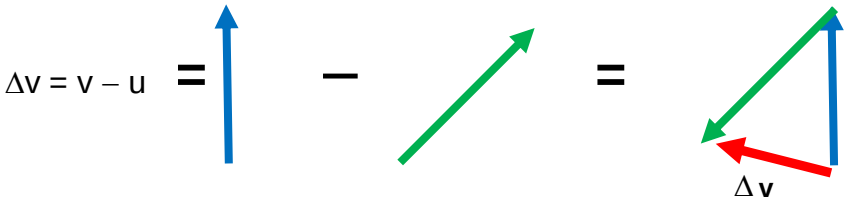
Consider a mass **m** travelling at a constant speed in a horizontal circle. In the position shown below the instantaneous velocity is shown as a vector **u** in a direction North East.



If we look at this mass a short time later it will look like that shown in the diagram below. The instantaneous velocity is now **v** in a direction North.



Thus, the change in velocity Δv is given by



The change in velocity Δv is shown above. Note that **Δv** must always act radially inwards. Therefore, the mass is accelerating even though the speed is not changing. This acceleration is called **radial or centripetal** (centre-seeking) **acceleration,** indicating that the acceleration always acts towards the centre of the circle**.**

Radial acceleration describes the change in velocity with time for an object moving in a circular path at constant speed. The acceleration is given by:

$$a = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2} = 4\pi^2 \, r \, f^2$$

a = centripetal acceleration (ms^{-2}), v = speed (m/s).
Note: v = (2×π×r)/T or Circumference Travelled ÷Time
r = radius of circle (m), T = period of motion (s)
f = frequency of motion (Hz) **Note: *f* =1/T**

Radial (Centripetal) Force

As stated in Newton's 2nd Law of Motion, the net or unbalanced force on an object is given by $\Sigma F = m \times a$, where the net force is always in the same direction as the acceleration. Hence, the net force – as with the centripetal acceleration – acts towards the centre of the circle. This net force is called the radial or **radial** or **centripetal force F_c.** Given that $\Sigma F = m \times a$

$$F_C = m \times a = m \times \frac{v^2}{r} = m \times \frac{4\pi^2 r}{T^2} = m \times 4\pi^2 \, r \, f^2$$

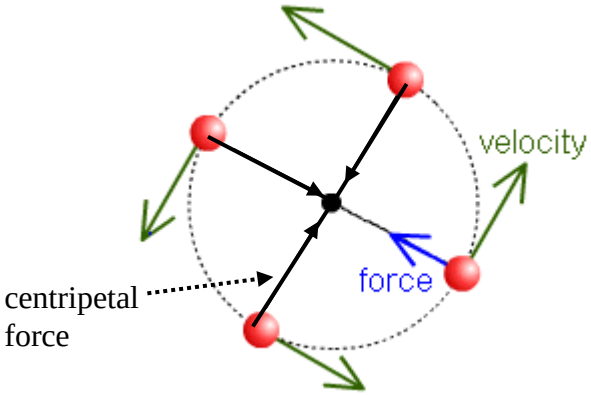
Using our relationships between linear and angular motion we have $v = r\,\omega$. Hence

$$F_C = m \times \frac{v^2}{r} = m \times \frac{r^2\omega^2}{r} = m\,r\,\omega^2$$

In angular terms:

$$F_C = m\,r\,\omega^2$$

An object travelling in a circle at constant speed will have a radial force that always acts towards the centre (see below), while the velocity is tangential to the circle. Note here that the force is always at right angles to the velocity. In the case of the hammer thrower (see example on next page) the radial force is being provided by the thrower at the centre of the circle. This force is a tensional force.



Forces that cause circular motion

In the hammer thrower example below the centripetal force was provided by an inwards **tension** that the thrower was exerting on the hammer. There are other forces that cause circular motion. Always try to identify the force causing the object to turn in a circle.

For example, a car moving in a circle does so because the radial force is due to the **friction** between the tyres and the road.

The radial force causing a satellite (such as the Earth's natural satellite, the moon) to orbit the Earth is caused by the **gravitational force** of the Earth acting on the satellite.

Example

(Millar, G, et. al, 2000, *Physics 12,* Heinemann)

An athlete swings a 2.5kg hammer in a horizontal circle of radius 0.8m at 2.0 revolutions per second.

(a) What is the period of rotation of the ball?
(b) What is the orbital speed of the ball?
(c) Calculate the radial acceleration of the ball.
(d) What is the magnitude of the net force acting on the ball?
(e) Name the force that is responsible for the radial acceleration of the ball?
(f) Describe the motion of the ball if the wire breaks.

Solution

(a) $T = 1/f$ where f is the frequency of the hammer (2 rev s^{-1})
$= 1/2.0\ s^{-1} = 0.50\ s$

(b) $v = (2\times\pi\times r)/T = 2\pi fr$ Recall: $f = 1/T$
$= 2\pi(2.0\ Hz)(0.80\ m) = 10.1\ m\ s^{-1}$

(c) $a = v^2/r$
$= (10.053\ m\ s^{-1})^2/0.80\ m = 126\ m\ s^{-2}$ towards centre of circle

(d) $F = ma$
$= (2.5\ kg)(12.6\ m\ s^{-2}) = 316\ N$

(e) The force that is responsible for the radial acceleration of the ball is the tension in the wire, which is directed radially inwards at all times.

(f) If the wire breaks, the ball will move off at a tangent to the circle with a speed of 10.1 m s^{-1}.

Exercise

Take g = 9.81ms^{-2}

1 The moon's radial acceleration towards the Earth is 2.73 $\times10^{-3}\ ms^{-2}$. If the moon's orbit about the Earth is 3.85×10^{8} m, calculate the speed of the moon in its orbit around the Earth. Give your answer to two decimal places.

2 An ice skater of mass 50 kg is skating in a horizontal circle of radius 1.5m at a constant speed of 2 m/s. (a) What is the radial acceleration of the skater? (b) What is the radial force acting on the skater? Give your answers to one decimal place.

3 A 10 g cork is swung in a horizontal circle with a radius of 50 cm. It makes 48 revolutions in 12 seconds. What is the tension in the string? Give your answer to one decimal place

4 A car of mass 800kg is taking a curve, of radius 60m, on a horizontal stretch of road at 60km/h. Will the car skid if the coefficient of friction is (a) 0.7 (b) 0.4? Explain.

5 The drum of a top-loading spin drier of radius 20 cm rotates in a horizontal plane at 2000 rpm. Calculate, to one decimal place (a) the speed and (b) the radial force of a 100g shirt attached to the inside of the drum wall.

6 Calculate (a) the speed and (b) radial acceleration of a person standing on the Earth's equator. The radius of the Earth is 6.38×10^{6} m. Give your answers to two decimal places.

Answers

1. 1.02 $\times10^{3}\ ms^{-1}$ 2. (a) 2.7ms^{-2} (b) 133N 3. 12.6N 4. (a) no (b) yes 5. (a) 41.9m/s (b) 6.877.3N 6 (a) 464.00 ms^{-1} (b) 0.03ms^{-2}

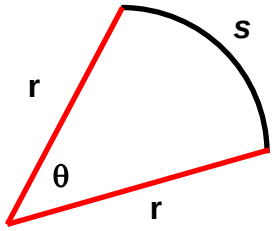
ANGULAR MOTION AND MOMENT OF INERTIA

Angular motion

Arc Length s (m)

Generally, the angular displacement θ at the centre of a circle is defined in radians by the equation

$$\theta = \frac{\text{arc length}}{\text{radius}} = \frac{s}{r} \quad \text{or} \quad \boxed{s = r\,\theta}$$



In a complete circle

$360^\circ = 2\pi$ radians or π *radians* $= 180^o$

Conversions

From radians to degrees: $\times \frac{180}{\pi}$ From degrees to radians $\times \frac{\pi}{180}$

Angular velocity ω (rad/s)

Angular Velocity ω is the rate of change of angular displacement

$$\boxed{\omega = \frac{\theta}{t}}$$

Linear and angular velocity are related by the formula

$$\boxed{v = r\,\omega}$$

Revolutions per Minute rpm

Since 1 revolution = 2π radians , I revolution per minute (60 seconds) is $\frac{2\pi}{60} = \frac{\pi}{30}$ rad/s

Conversions: From rpm to radians per second $\times \frac{\pi}{30}$ From radians per second to rpm $\times \frac{30}{\pi}$

Example A weight on the end of a string describes a circular path of radius 0.5m. If its linear velocity is 2m/s what is the angular velocity in (a) rad/s (b) rpm?

(a) $\omega = \frac{v}{r} = \frac{2}{0.5} = 4$ rad/s (b) $4 \text{ rad/s} = 4 \times \frac{30}{\pi} = 38.2$ rpm

Angular Acceleration α (rad/s²)

Angular acceleration is the rate of change of angular velocity, or $\boxed{\alpha = \frac{\omega}{t}}$

Linear and angular acceleration are related by the formula

$$a = r\,\alpha \qquad \text{or} \qquad a = \omega^2 r$$

Uniform acceleration: Equations of Angular Motion

It can be shown that equations derived for linear motion apply equally to angular motion

Linear Equations	*Angular Equations*
$s = v\,t$	$\theta = \omega\,t$
$a = \dfrac{v - v_0}{t}$	$\alpha = \dfrac{\omega - \omega_0}{t}$
$v = v_0 + at$	$\omega = \omega_0 + \alpha t$
$v^2 = v_0^2 + 2as$	$\omega^2 = \omega_0^2 + 2\alpha\theta$
$s = v_0 t + \frac{1}{2} a t^2$	$\theta = \omega_0 t + \frac{1}{2} \alpha t^2$

Example A flywheel increases in velocity from rest to 400 rpm. in 15 seconds. Calculate (a) the angular acceleration, and (b) the number of revolutions it makes in this time.

(a) $\omega_0 = 0,\ \omega = 400\,\text{rpm} = \dfrac{400 \times \pi}{30} = 41.9\ \text{rad/s},\ t = 15,\ a = ?$

Use equation $\omega = \omega_0 + \alpha t,$ $\qquad \alpha = \dfrac{\omega - \omega_0}{t} = \dfrac{41.9 - 0}{15} = 2.79\,\text{rad/s}^2$

(b) $\theta = ?$ Use equation $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$$\theta = \omega_0 t + \tfrac{1}{2}\alpha t^2 = 0(15) + \tfrac{1}{2}(2.79)(15)^2 = 314 \text{ radians} = \frac{314}{2\pi} = 50 \text{ revs}$$

Moment of Inertia

Net force, mass and acceleration in linear motion have a rotational analogue – net torque, mass moment of inertia and angular acceleration respectively. Moment of inertia is a measure of the resistance offered by an object to rotational movement, e.g. the bending moment in a beam.

Linear Motion	**Rotational Motion**
$\Sigma F = m\,a$	$\Sigma\,\tau = I\,\alpha$

where $\Sigma\tau$ = net torque (Nm); I = mass moment of inertia (kgm^2); α = angular acceleration (rad s^{-2}).

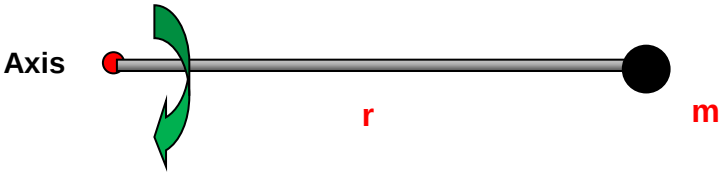
Example
Calculate the moment of inertia of a flywheel that has a net torque of 15 Nm and an angular acceleration of 25 rads^{-2}

Solution

$$\Sigma\tau = I \times \alpha \Rightarrow I = \frac{\Sigma\tau}{\alpha} = \frac{15}{25} = 0.6\,\text{kgms}^{-2}$$

The moment of inertia I of a rigid body m about an axis of rotation r is

$$\boxed{I = m\,r^2}$$



For two or more objects m_1, m_2, etc, the moment of inertia is given by the sum of the individual moments of inertia, or

$$\boxed{\Sigma mr^2 = m_1 r_1^2 + m_2 r_2^2}$$

Different objects have different moments of inertia, assuming uniform composition. For a flywheel (disc) $I = \frac{1}{2} mr^2$, a cylinder $I = \frac{1}{2} mr^2$, a thin hoop $I = mr^2$, a rod with its axis of rotation at one end $I = (1/3)\, mr^2$, and a rod with its axis of rotation at its centre $I = (1/12)\, mr^2$.

Rotational and Translational Kinetic Energy

An object rotating about an axis has rotational as well as translational kinetic energy. We know that $KE = \frac{1}{2} mv^2$. Since $v = r\omega$, we have $KE = \frac{1}{2} mr^2\omega^2$, or

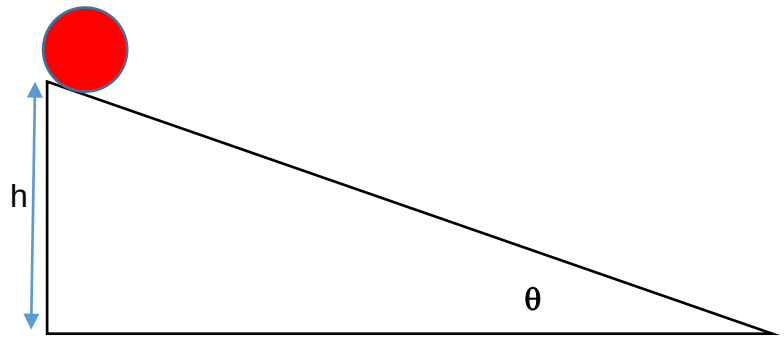
$$\boxed{KE = \tfrac{1}{2}\, I\, \omega^2}$$

where I is the moment of inertia ($I = mr^2$) of the object and ω is its angular velocity. For example, a wheel rolling down a hill has both translational and rotational kinetic energy given by

$$\boxed{KE = \tfrac{1}{2}\, m\, v^2 + \tfrac{1}{2}\, I\, \omega^2}$$

where v is the linear velocity and ω is the angular velocity.

We can use the conservation of energy to analyse an object's rotational, translational and potential energy at different points in its motion, as shown below. For instance, consider a sphere of mass m, moment of inertia I, and situated at the top of a ramp of height h metres. It rolls down the ramp with velocity v and angular velocity ω.



Its total energy TE at this position, and at any other position in its motion down the ramp is given by

$$\boxed{TE = \tfrac{1}{2}\, m\, v^2 + \tfrac{1}{2}\, I\, \omega^2 + m\, g\, h}$$

While the value of the total energy of the object remains the same at all times from the law of conservation of energy, the proportions of the object's rotational, translational and potential energy components will change. See Question 9 in the Exercise at the end of this worksheet.

1 Calculate the angle (a) in radians (b) in degrees (to 1 decimal point), at the centre of a circle of radius 500mm with arc length 1m.

2 Calculate (a) the angular velocity and (b) the linear velocity of the seconds hand of a clock. The seconds hand is 10cm long. Give your answers to 3 decimal places.

3 Calculate the angular velocity (to 3 decimal places) of a car rounding a corner of radius 100m travelling at 60km/h.

4 Calculate the moment of inertia of a 500g rod of length 155cm with (a) its axis of rotation at one end and (b) with its axis of rotation at its centre. The moment of inertia of a rod with its axis of rotation at one end is $I = \frac{1}{3} mr^2$, and with the axis of rotation at its centre $I = \frac{1}{12} mr^2$.

5 The rotational speed of a CD, of diameter 120mm, is 210 rpm. Calculate (a) the linear speed and (b) the acceleration of a bit of dirt on its outer edge.

6 A flywheel increases in velocity from rest to 300 rpm. In 10 seconds. Calculate (a) the angular acceleration, and (b) the number of revolutions it makes in this time.

7 Two beads (spheres) of mass 0.2kg and 0.3kg are threaded on to a straight wire, 2 metres apart. The 0.2 kg bead is situated to the left of the 0.3kg bead. Calculate the moment of inertia of the system of beads and wire if they are rotated about an axis situated about an axis
   (a) situated halfway between the beads.
   (b) situated 0.5m to the left of the 2kg bead.

8 Calculate the acceleration of a flywheel of mass 500g and diameter 80cm if the torque applied to it is 1.2Nm.

9 Refer to the diagram of the object at the top of a ramp in the Rotational and Translational Kinetic Energy section above . The sphere has a mass M, radius R and its moment of inertia is

$$I = \frac{2}{5} MR^2$$

.

Show that the speed of the sphere at the bottom of the ramp, if it starts from rest, is given by

$$v = \sqrt{\frac{10}{7} g\, h}$$

Answers

1(a) 2 radians (b) 114.6° 2. (a) 0.105 rad/s (b) 0.011 m/s 3. 0.167 rad/s 4. (a) 0.40 kgm^2 (b) 0.10 kgm^2

5.(a) 2.6m/s (b) 0.6 ms^{-2} 6(a) 3.1 rads^{-2} (b) 25 revs 7 (a) 0.5kgm^2 (b) 0.85kgm^2 8. 30 rad/s

ELECTRICITY: BASIC CONCEPTS

Current I and Charge q

Current (I) is the rate of flow of electric charge (q) past a point.

$$\text{Current} = \frac{\text{Charge}}{\text{Time}} \qquad \text{or} \qquad I = \frac{q}{t}$$

Units: Current is measured in Amps (A). Charge is measured in Coulombs (C). Time is measured in seconds (s)

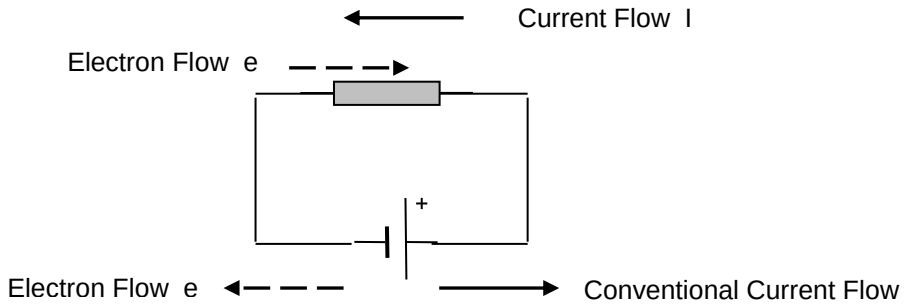
Example 1

Determine the charge that has flowed through a torch battery producing a current of 300mA if it has been left on for 20 minutes.

If $I = q/t$, $q = I \times t$ where $I = 300 \times 10^{-3}$A, and $t = 20 \times 60 = 1200$s.
Thus, $q = 0.300 \times 1200 = 360$C

A circuit must be closed (complete) for current to flow. Contrary to popular belief the current does not get weaker as it goes around the circuit. The direction of the current is taken to be from the positive terminal of the battery, around the circuit, to the negative terminal of the battery. This is called conventional current flow. In the diagram below conventional current flow is counter clockwise.

In actual fact the electrons are the charges that move, and they flow from the negative terminal of the battery, around the circuit, to the positive terminal of the battery, ie, in the opposite direction to conventional current flow. In the diagram below electron flow is clockwise.



In terms of electron charges:

$$1\,\text{coulomb of charge}\quad 1C = 6.25\times10^{18}\,\text{electrons} \qquad \text{or} \qquad 1\,\text{electron}\quad 1e = 1.6\times10^{-19}\,C.$$

EMF (Electromotive Force)

If charges are moving they have energy. A battery or generator is the usual source of this energy. When electrons flow through a battery or other power supply they gain electrical potential energy. As the electrons flow around a circuit, they lose this energy when passing through components that have resistance, e.g. globes.

The energy transferred to one coulomb of charge within the battery is called the electromotive force EMF of the battery.

The unit of EMF is the volt (V). It can be considered to be a concentration of charge. The energy, measured in Joules (J), required in pushing the charges together is stored as electrical potential energy. A battery with an EMF of 6V, for instance, transfers 6 Joules of energy to each coulomb of charge.

The energy transferred E to a circuit component per unit charge q is called the **potential difference** PD between the two terminals of the component. The unit of potential difference is also the volt.

$$\text{EMF} = \frac{\text{Energy}}{\text{Charge}} \quad \text{or} \quad V = \frac{E}{q}$$

Units: EMF is measured in Volts (V). Charge is measured in Coulombs (C). Energy is measured in Joules (J).

Note: The terms EMF and PD are often simply referred to as voltage.

Since $E = V \times q$, and $q = I \times t$

$$E = V \times I \times t$$

Example 2

The potential difference across a torch globe is found to be 2.7V. The current flowing through it is 0.2A.

(a) How much charge flows through the torch in 1 minute?

(b) How much energy is lost by this charge?

(a) $q = I \times t = 0.2 \times 60s = 12C$

(b) Each coulomb lost 2.7J of energy (why?).

Since $V=E/q$, $E = q \times V = 12 \times 2.7 = 32.4J$

Power P

Power (P) is the rate of energy transfer, ie. energy transfer divided by the time taken.

$$\text{Power} = \frac{\text{Energy}}{\text{Time}} \quad \text{or} \quad P = \frac{E}{t} = \frac{V \times I \times t}{t} = V \times I$$

Units: Power is measured in Joules per second (J/s) or Watts (W).

Exercise

5. Calculate the amount of charge that would flow through a:
   (a) 5mA pocket calculator in 10 minutes.
   (b) 200A car starter motor in 5 seconds.
   (c) 400mA light bulb in 1 hour.

6. 1.7×10^{26} electrons pass a point in a wire each second. What electric current does this represent?

7. A charge of 5C flows from a battery through an electric water heater and delivers 100J of heat to the water. What was the EMF (voltage) of the battery?

8. How much energy will each coulomb of charge flowing from a 9V MP3 player possess?

5. How much charge must have flowed through a 12V car battery if 2kJ of energy was delivered to the starter motor?

7. What is the power used by a:
   (a) 3V torch bulb drawing 0.2A?
   (b) Starter motor which takes 200A from a 12V battery?
   (c) Mains-powered (240V) toaster rated at 3A?

8. How much current is used by a:
   i. 60W, 240V light bulb? Give your answer in milliamps (mA).
   ii. 1200W mains-powered (240V) heater?
   iii. 90W car (12V) windscreen wiper motor?

9. What is the voltage of a:
   i. 100W spotlight which draws 4A?
   ii. 200mW radio operating with a current of 23mA?
   iii. 7500W industrial motor using 18A?

9. A large power station generator might be rated as 500MW with a 24kV output. What current would it be generating? Give your answer in kiloamps (kA).

Answers

Exercise

1. (a) 3C (b) 1000C (c) 1440C 2. 2.7×10^{7}A 3. 20V 4. 9J 5. 167C

6. (a) 0.6W (b) 2400W (c) 720W 7. (a) 250mA (b) 5A (c) 7.5A 8.(a) 25V (b) 8.7V

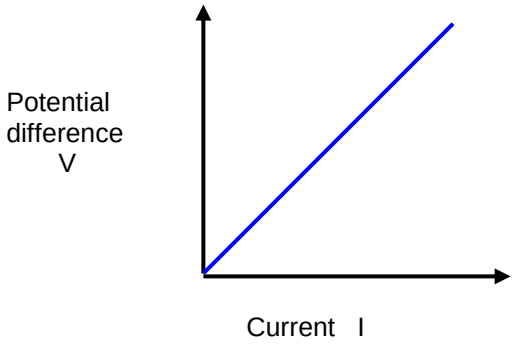
(c) 417V 9. 21kA

ELECTRICITY: OHM'S LAW

Resistance R

Under constant physical conditions the potential difference (change in voltage) is proportional to the current ($V \propto I$), and is given by Ohm's Law:

$$V = I \times R$$



Ohmic Conductors

R (the slope of the V-I graph above) is a constant for a given conductor under these conditions and is called the resistance. (Unit: Ohm Ω). The circuit element is said to be ohmic, commonly known as an **ohmic conductor**.

$$\text{Since } P = VI \quad \text{and} \quad V = IR, \quad P = I^2R = \frac{V^2}{R}$$

Example 1

An ohmic conductor of 5Ω is supplied with a voltage which can vary from 1V to 100V.
(a) What will be the range of current that will flow in it?
(b) How much energy will be dissipated (used up) in the resistor each second?

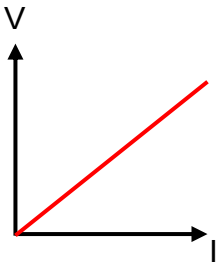
(a) For 1V: $V=I \times R$, $I=V/R = 1/5 = 0.2A$; For 100V: $V=I \times R$, $I=V/R = 100/5 = 20A$
(b) At 1V, 0.2A means that 0.2C flow through the resistor each second. Recall: $I=q/t$.
Therefore, 1V: $E=V \times q = 1 \times 0.2 = 0.2J$.
Similarly, at 100V, $E = Vq = 100 \times 20 = 2000J$

Non-Ohmic Conductors

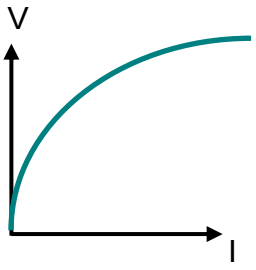
Devices that have a constant resistance, as described above, are known as ohmic conductors. The graph of V versus I is a straight line that must go through the origin. If the graph is not a straight line then the device is called a **non-ohmic conductor**. See graphs below.

Light bulbs such as car headlamps are common examples of non-ohmic conductors. As the voltage increases, the current will not increase in proportion, as shown in the graph below.

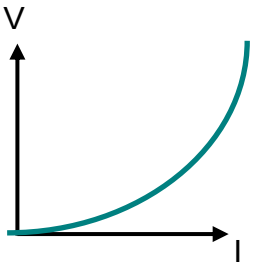
Other non-ohmic conductors include devices whose resistance changes with light or temperature. These are particularly useful as detectors in sensors which need to respond to changes in light (see diode graph below) or temperature levels.



Ohmic Conductor
eg, resistor

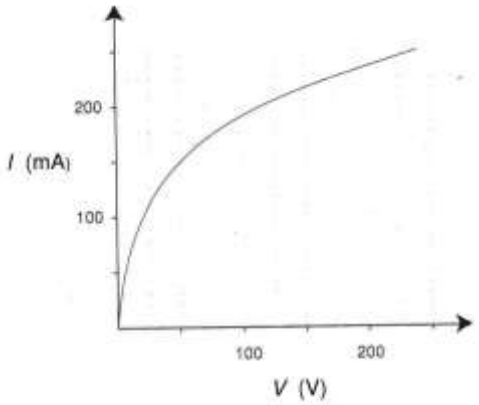


Non-Ohmic Conductor
eg, car headlamp bulb



Non-Ohmic Conductor
eg, diode

Example 2



The graph at left represents the I-V characteristics of a non-ohmic device. What is the resistance at
(a) 50V (b) 150V (c) 200V

Note: This is an I-V graph, not a V-I graph. The slope at any point would be the **inverse** of the

Resistance is given by R = V/I **at any point on the graph**. Note that the current is given in mA (100mA = 0.1A, since 1A = 1000mA).

(a) At 50V, R = 50V/150mA = 50/0.15 = 333.3Ω
(b) At 150V, R = 150V/210mA = 150/0.21 = 714.2Ω
(c) At 200V, R = 200V/240mA = 200/0.24 = 83.3Ω

Resistivity ρ

The resistance of a wire is a measure of the ability of the wire to impede the flow of electrons along its length. It can be shown that the resistance:

- increases if the wire is longer, L.
- decreases if the wire is thicker, which means it has a larger cross-sectional area, A.
- depends on the metal used for the wire, ρ (rho).

These can be summarised in the formula:

$$R = \frac{\rho L}{A}$$

resistance R is measured in Ω,
resistivity ρ (rho) is measured in Ωm, and depends on the metal used
L is measured in metres m

Example 3

Normal household wiring uses 1.8mm diameter copper wire. The resistivity of copper is $\rho = 1.7 \times 10^{-8}\ \Omega m$.

(a) What is the resistance of a 10m long piece of this copper wire?

(b) What voltage drop will there be along it if a current of 10A is flowing through it?

(a) The cross-sectional area (which is a circle) of the wire is given by:

$A = \pi \times r^2 = 3.14 \times (0.9 \times 10^{-3})^2 = 2.5 \times 10^{-6}\ m^2$. Note: mm is converted to metres.

$R = \rho L/A = 1.7 \times 10^{-8} \times 10/(2.5 \times 10^{-6}) = 0.068\Omega$

If the wire is carrying a 10A current there will be a voltage drop of:

$V = I \times R = 10 \times 0.068 = 0.68V$ along the length of the wire.

Exercise

1. A student finds that the current through a resistor is 3.5A while a voltage of 2.5V is applied to it.
   (a) What is the resistance?
   (b) The voltage is then doubled and the current is found to increase to 7.0A. Is the resistor ohmic or not?

2. Rose and Rachel are trying to find the resistance of an electrical device. They find that at 5V it draws a current of 200mA and at 10V it draws a current of 500mA. Rose says that the resistance is 25Ω, but Rachel maintains that it is 20Ω. Who is right and why?

3. Nick has an ohmic resistor to which he has applied 5V. He measures the current at 45mA. He then increases the voltage to 8V. what current will he find now?

4. Lisa finds that when she increases the voltage across an ohmic resistor from 6V to 10V the current increases by 2A.
   (a) What is the resistance of the resistor?
   (b) What current does it draw at 10V?

5. The resistance of a certain piece of wire is found to be 0.8Ω. What would be the resistance of:
   (a) a piece of the same wire twice as long?
   (b) a piece of the same wire double the thickness?

6. If the resistance of a copper wire 20m long and 1mm in diameter is 0.44Ω, what will be the resistance of the same length of wire 2mm in diameter?

Answers

Exercise

2. (a) 0.71Ω (b) ohmic 2. Both right at different voltages 3. 72mA 4. (a) 2Ω (b) 5A
5. (a) 1.6Ω (b) 0.2Ω 6. 0.11Ω.

ELECTRICITY: SERIES CIRCUITS

Kirchoff's Laws

The principle of conservation of charge and the principle of conservation of energy can be used to establish two very important rules that apply to all electric circuits.

> In any electrical circuit the sum of all currents flowing into any point is equal to the sum of the currents flowing out of it.

Example 1

If at any junction of three wires there is 2A flowing in on one wire and 3A flowing in on another, then there must be a current of 5A flowing out on the third.

Sometimes this rule is abbreviated simply to 'the sum of all currents at a point is zero'. Remember that currents flowing into the point will be positive and those flowing away from the point negative.

> The total potential drop around a closed circuit must be equal to the total EMF in the circuit.

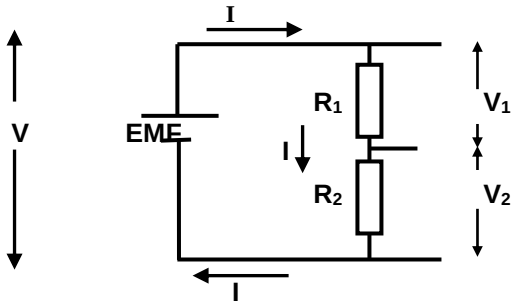
Example 2

If we know that a torch battery supplies an EMF of 3.0V and we measure a 2.8V PD (potential difference) across the bulb, there must be a 0.2V drop somewhere else in the circuit – possibly across the switch contacts if they are a little dirty.

Two ways of connecting circuits

No matter how complex a circuit, it can always be broken up into sections in which circuit elements are combined either in series or in parallel: that is, one after another, or beside each other respectively. The remainder of this worksheet studies Series Circuits. The following worksheet studies Parallel Circuits.

Series Circuits



The sum of all the potential differences across the resistors (V_1 and V_2) will equal the potential difference, or EMF, of the supply, which in this case is V.

The EMF of the battery is therefore given by

$$\text{EMF} = V = V_1 + V_2$$

For resistors in series the equivalent resistance is equal to the sum of the individual resistances, or:

$$R_{EQ} = R_1 + R_2$$

Where R_{EQ} is one resistance equivalent to the sum of R_1 and R_2.

Given that the current I leaving the battery **must** flow through both R_1 and R_2, then the current flowing in a series circuit must always be the same, or:

For resistors connected in series the same current must pass through each resistor.

Example 3

Two pieces of nichrome wire (as used in heater elements) have resistances of 10Ω and 20Ω.

(a) What current would flow through them, and what power will be produced in them, if they are separately connected to a 12V battery?

(b) If they are connected in series what is their total resistance?

(c) When placed in series across the 12V battery, what current will flow through them and what power will be produced?

Solution

(a) The current will be given by I=V/R, so for the two wires separately the currents will be 12/10 = 1.2A, and 12/20 = 0.6A. The power is found from P=V/I and so will be 12/1.2 = 14.4W and 12/0.6 = 7.2W.

(b) When connected in series the total resistance will be 10 + 20 = 30Ω

(c) The current that flows from the 12V battery will be I=V/R = 12/30 = 0.4A. The total power will be V/I = 12/0.4 = 30W.

Exercise

7. Emily has found a point in her car where four wires are attached together. She finds currents of +2.5A and +1.0A in two of the wires, and -4.2A in a third ('+' means current into the point and '–' means current out of the point). What is the current in the fourth wire?

8. Two torch bulbs are placed in series with each other and a 4.5V battery. The current through one is found to be 0.25A and voltage across it is 2.1V.
   (a) What is the current through the other bulb? Give your answer in mA.
   (b) What is the voltage across the other bulb?

9. Bill has bought two 12V headlamps for his truck but finds that it has a 24V battery. He decides that the simplest way to overcome the problem is to wire them in series.
   (a) Will the headlamps work correctly?
   (b) Do you see any problems with this scheme?

10. Two equal resistors are placed in series and found to have a combined resistance of 34Ω. What is the resistance of each one?

11. A 10V power supply is used across two separate resistors. The current through one is found to be 0.4A, and through the other 0.5A. When they are combined in series, what current (in mA) will flow through them and what is their effective resistance?

12. A 400Ω resistor and a 100Ω resistor are placed in series across a battery with an EMF of 5V.
   (a) How much current will flow from the battery? Give your answer in mA.
   (b) What will be the voltage across each resistor?

Answers

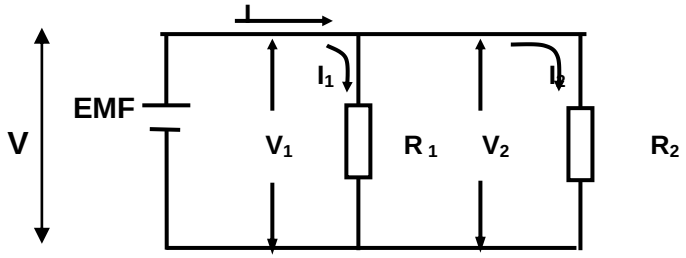
Exercise

1. +0.7V 2. (a) 250mA (b) 2.4V 3. (a) yes (b) yes. If one of the headlamps blows, then the other one will turn off as they are both in series.
4. 17Ω 5. 220mA, 45Ω 6. 10mA, 4V and 1V.

ELECTRICITY: PARALLEL CIRCUITS

When ohmic resistors are placed in parallel (side by side) with each other we can expect a greater current to flow at any particular voltage. In fact the current will be the sum of the two separate currents. We thus expect the effective resistance of two resistors in parallel to be somewhat less than either of them alone.

Consider two resistors R_1 and R_2 connected in parallel across a battery



If the two resistors are placed in parallel it is the sum of the two currents I_1 and I_2 that must add to get the total current I leaving the battery:

$$I = I_1 + I_2$$

When resistors are connected in parallel the potential difference across each resistor V_1 and V_2 is the same as the EMF of the battery, or

$$EMF = V = V_1 = V_2$$

For two resistors R_1 and R_2 in parallel:

$$\frac{1}{R_{EQ}} = \frac{1}{R_1} + \frac{1}{R_2}$$

where R_{EQ} is one resistance equivalent to the two resistors.

Example 1

Two pieces of nichrome wire are found to have resistances of 10Ω and 20Ω.

(a) If they are connected in parallel what is their effective resistance?

(b) What total current will flow through them and what power will be produced if the combination is placed across a 12V battery?

(a) The effective resistance is found from $\frac{1}{R_{EQ}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{10} + \frac{1}{20} = \frac{3}{20}$

Thus, $R_{EQ} = 20/3 = 6.7\Omega$
Note how the equivalent resistance is less than the two resistances R_1 and R_2.

(b) The total current is given by $I = V/R = 12/6.7 = 1.8A$. The power is therefore
$P = V \times I = 12 \times 1.8 = 21.6W$

Exercise

13. Two torch bulbs are placed in parallel with each other across a 3.0V battery. The current through the battery is 0.55A. The current through one of the bulbs is 0.25A.
    (a) What is the current through the other bulb?
    (b) What is the voltage across the other bulb?

14. What is the effective resistance of two 10Ω resistors in (a) series, (b) parallel?

15. Two equal resistors are placed in parallel and found to have a combined resistance of 34Ω. What is the resistance of each one?

16. A 10V power supply is used across two separate resistors. The current through one is found to be 0.4A, and through the other 0.5A. When they are combined in parallel what current will flow through them and what is their effective resistance?

17. A current of 3A is found to be flowing through two resistors of 20Ω and 10Ω in parallel.
    (a) What is the effective resistance of the combination?
    (b) What is the voltage across the pair of resistors?
    (c) How much current will be flowing in each resistor?

18. How much power will be dissipated (used up) in each of the two resistors in Question 5?

19. Three resistors of 900Ω, 1.5kΩ and 2.0kΩ are to be used in a circuit. What is their effective resistance if they are all placed (a) in series, (b) in parallel?

Answers

Exercise

1(a) 0.3A (b) 3V 2(a) 20Ω (b) 5Ω 3. 68Ω 4. 0.9A, 11Ω 5(a) 6.7Ω (b) 20V (c) 1A (in 20Ω) and 2A 6. 20W (in 20Ω), 40W 7(a) 4.4kΩ (b) 0.44kΩ