RMIT UNIVERSITY

# LT5 LAPLACE TRANSFORMS – ELECTRICAL CIRCUITS

## Applying Laplace Transforms to Electrical Circuits

### *Example*

For the LRC circuit given by

$$\mathrm{L}\frac{di}{dt} + Ri + \frac{1}{C}\int i\ dt = E_0 \qquad \text{where } \frac{dq}{dt}(0) = \mathrm{q}(0) = 0$$

And $\mathrm{L} = 1, \mathrm{R} = 3, \mathrm{C} = 0.5, E_0 = 10$

Find:

(a) the charge $q(t)$ on the capacitor

(b) the resulting current $i(t)$ in the circuit, using Laplace transforms

**Solution**

$$\mathrm{L}\frac{di}{dt} + Ri + \frac{1}{C}\int i\ dt = E_0$$

Since $i = \frac{dq}{dt}$ we can write this equation as

$$\mathrm{L}\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = E_0$$

Since $\mathrm{L} = 1, \mathrm{R} = 3, \mathrm{C} = 0.5, E_0 = 10$ this equation becomes

$$\frac{d^2q}{dt^2} + 3\frac{dq}{dt} + 2q = 10$$

Recall that

$$\mathcal{L}\left[\frac{d^2}{dt^2}q(t)\right] = s^2Q(s) - s.q(0) - q'(0)$$

$$\mathcal{L}\left[\frac{d}{dt}q(t)\right] = sQ(s) - q(0)$$

$$\mathcal{L}[q(t)] = Q(s)$$

$$\mathcal{L}[a] = \frac{a}{s}$$

So if $\frac{d^2q}{dt^2} + 3\frac{dq}{dt} + 2q = 10$

Then $\mathcal{L}\left[\frac{d^2q}{dt^2} + 3\frac{dq}{dt} + 2q\right] = \mathcal{L}[10]$

And $[s^2Q(s) - s.q(0) - q'(0)] + 3\,[sQ(s) - q(0)] + [2Q(s)] = \frac{10}{s}$

Therefore $s^2Q(s) + 3sQ(s) + 2Q(s) = \frac{10}{s}$

And since we are given that $q(0) = 0 \text{ and } q'(0) = 0$

$\Rightarrow \qquad Q(s)\,[s^2 + 3s + 2] = \frac{10}{s}$

$$\Rightarrow \qquad Q(s) = \frac{10}{s\,[s^2 + 3s + 2]}$$

$$Q(s) = \frac{10}{s\,(s+1)(s+2)}$$

We can now resolve this into partial fractions

Let $\frac{10}{s\,(s+1)(s+2)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+2}$

$$\Rightarrow \qquad 10 = A(s+1)(s+2) + B(s)(s+2) + C(s)(s+1)$$

Setting $s = 0$: $10 = 2A$ $\Rightarrow$ $A = 5$

Setting $s = -1$: $10 = -B$ $\Rightarrow$ $B = -10$

Setting $s = -2$: $10 = 2C$ $\Rightarrow$ $C = 5$

So $\qquad Q(s) = \frac{10}{s\,(s+1)(s+2)}$

Can now be written as: $Q(s) = \frac{5}{s} - \frac{10}{s+1} + \frac{5}{s+2}$

Now taking the inverse Laplace transform:

$$q(t) = \mathcal{L}^{-1}\left[\frac{5}{s} - \frac{10}{s+1} + \frac{5}{s+2}\right]$$

$$q(t) = 5 - 10e^{-t} + 5e^{-2t}$$

To find the current $i(t)$:

$$i(t) = \frac{dq}{dt} = \frac{d}{dt}(5 - 10e^{-t} + 5e^{-2t}) = 10e^{-t} - 10e^{-2t}$$

Note: This problem could also have been solved using a 2nd order differential equation, without Laplace transforms had the question not specifically asked for its solution using Laplace transforms.