LT2 LAPLACE TRANSFORMS – TABLE, THEOREMS

Laplace Transforms Table & Operational Theorems

The fundamental rule for Laplace Transforms is:

$$L[f(t)] = F(s) = \int_0^\infty e^{-st} f(t)dt$$

Rather than perform what can be a complicated integration, a table is provided with some of the most common transforms already completed:

Transform Table:

$\boldsymbol{f(t)}$	$\boldsymbol{L[f(t)] = F(s)}$
$\delta(t)$ Impulse function	1
1	$\frac{1}{s}$
t^n	$\frac{n!}{s^{n+1}}$ $\quad n = 1, 2, 3, \ldots$
e^{at}	$\frac{1}{s-a}$
$\sin bt$	$\frac{b}{s^2+b^2}$
$\cos bt$	$\frac{s}{s^2+b^2}$
$\sinh bt$	$\frac{b}{s^2-b^2}$
$\cosh bt$	$\frac{s}{s^2-b^2}$

Laplace Transform Operational theorems:

$\boldsymbol{f(t)}$	$\boldsymbol{L[f(t)] = F(s)}$	
$af(t) + bg(t)$	$L[af(t) + bg(t)] = aF(s) + bG(s)$	$a, b \in \mathbb{R}$
$f(at)$	$L[f(at)] = \frac{1}{a}F(s)$	$L[f(t)] = F(s)$ $a > 0$
$e^{at}\, f(t)$	$L[e^{at} f(t)] = F(s - a)$	$L[f(t)] = F(s)$
$f(t - \tau)H(t - \tau)$	$L[f(t - \tau)H(t - \tau)] = e^{-\tau s}\, F(s)$	$L[f(t)] = F(s)$
$t^n\, f(t)$	$L[t^n\, f(t)] = (-1)^n \ \frac{d^n F}{ds^n}$	
$\frac{f(t)}{t}$	$L\left[\frac{f(t)}{t}\right] = \int_s^\infty F(u)du$	$L[f(t)] = F(u)$
$\frac{d}{dt}f(t)$	$L\left[\frac{d}{dt}f(t)\right] = sF(s) - f(0)$	
$\frac{d^2}{dt^2}f(t)$	$L\left[\frac{d^2}{dt^2}f(t)\right] = s^2F(s) - sf(0) - f'(0)$	

Exercise

Determine the Laplace Transform of the following expressions:

a. t^3 b. t^7 c. $\sin 4t$ d. e^{-2t}

e. $\cos\left(\frac{t}{2}\right)$ f. $\sinh 3t$ g. $\cosh 5t$ h. $t\sin 4t$

i. $e^{-t}\sin 2t$ j. $e^{3t}\cos t$

Answers

a. $\frac{6}{s^4}$ b. $\frac{7!}{s^8}$ c. $\frac{4}{s^2+16}$ d. $\frac{1}{s+2}$

e. $\frac{s}{s^2+0.25}$ f. $\frac{3}{s^2-9}$ g. $\frac{s}{s^2-25}$ h. $\frac{8s}{(s^2+16)^2}$

i. $\frac{2}{(s+1)^2+4}$ j. $\frac{s-3}{(s-3)^2+1}$

Exercise

Determine the Laplace Transform of the following expressions, given $y(0) = 3$ & $y'(0) = 1$.

a. y' b. y'' c. $3y'' - y'$

d. $y'' + 2y' + 3y$ e. $3y'' - y' + 2y$ f. $-4y'' + 5y' - 3y$

g. $3\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 8y$ h. $4\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 6y$

Answers

a. $sY - 3$ b. $s^2Y - 3s - 1$

c. $3s^2Y - sY - 9s$ d. $(s^2 + 2s + 3)Y - 3s - 7$

e. $(3s^2 - s + 2)Y - 9s$ f. $(-4s^2 + 5s - 3)Y + 12s - 11$

g. $3s^2Y + 6sY + 8Y - 9s + 21$ h. $4s^2Y - 8sY + 6Y - 12s + 20$

Exercise

Determine the inverse Laplace Transform of the following expressions using tables.

a. $\frac{1}{s}$ b. $\frac{2}{s^3}$ c. $\frac{1}{s+2}$

d. $\frac{5}{s-3}$ e. $\frac{2}{s^2+4}$ f. $\frac{s}{s^2+9}$

g. $\frac{2}{(s+1)^2}$ h. $\frac{12}{s^2-9}$ i. $\frac{6s}{s^2-8}$

j. $\frac{2}{(s+1)^2-4}$ k. $\frac{s+3}{(s+3)^2-4}$ l. $\frac{2}{(s+3)^2-4}$

m. $\frac{6s}{s^2-5}$

Answers

a. 1 b. t^2 c. e^{-2t}

d. $5e^{3t}$ e. $\sin 2t$ f. $\cos 3t$

g. $2te^{-t}$ h. $4\sinh 3t$ i. $6\cosh\sqrt{8}\,t$

j. $e^{-t}\sinh 2t$ k. $e^{-3t}\cosh 2t$ l. $e^{-3t}\sinh 2t$

m. $6\cosh\sqrt{5}\,t$