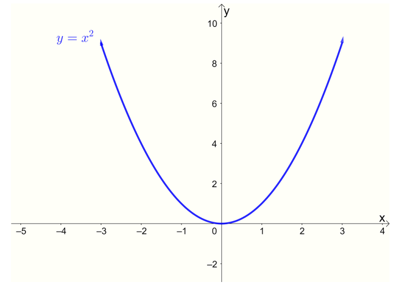
FG8 Quadratic Graphs



A quadratic graph is the graph of a quadratic function. This module describes the graphing of quadratic functions. A quadratic function has the form $y = ax^2 + bx + c$ where $a \neq 0$.

The graph of a quadratic function is called a parabola.

To sketch a parabola, find and label:

(a) the y-intercept (put $x = 0$)

(b) the x-intercepts (put $y = 0$)

(c) the vertex (turning point)

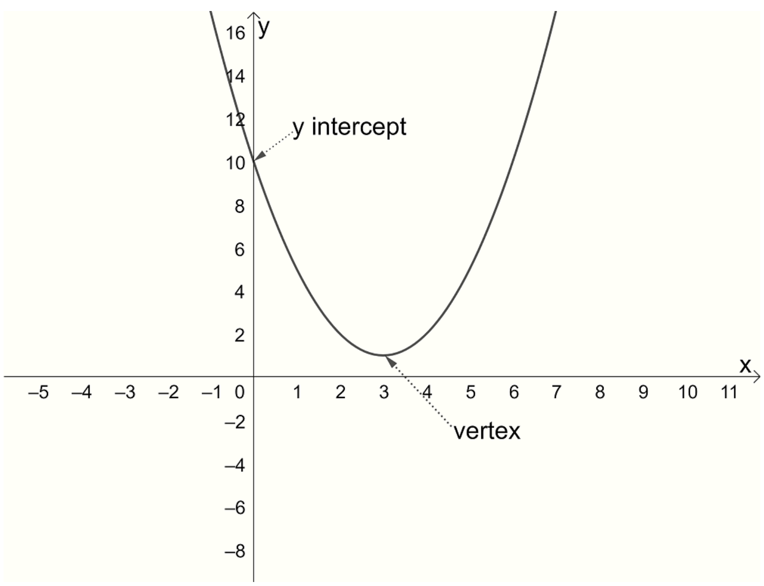
The co-ordinates of the vertex are given by:

x co-ordinate $\left(-\frac{b}{2a}\right)$

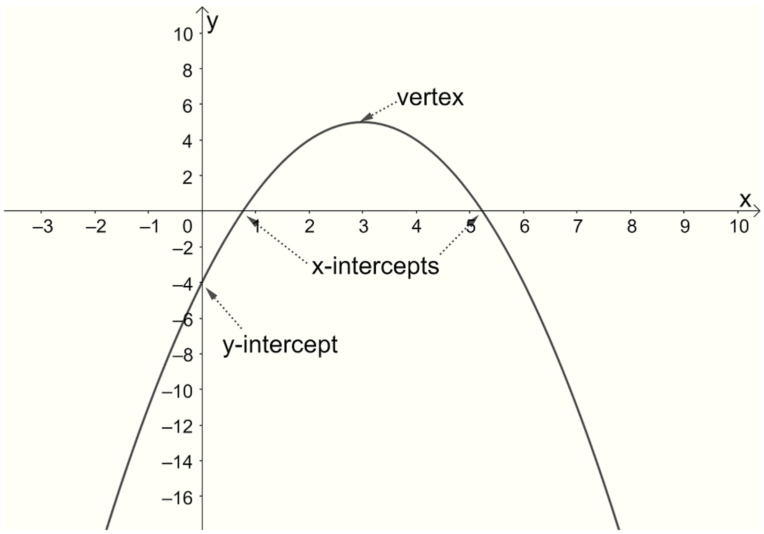
y co-ordinate: substitute the value of the x co-ordinate in the equation for y.

A parabola is symmetrical about a vertical line through the vertex.

If $a > 0$, then the parabola opens upwards (and has a minimum turning point).



If $a < 0$, then the parabola opens downwards (and has a maximum turning point).



A quadratic function may also be written in turning point form: $y = a(x-h)^2 + k$, where (h, k) is the turning point.

Examples

$y = (x-3)^2 + 4$ has a turning point at $(3, 4)$
$y = (x+5)^2 + 2$ has a turning point at $(-5, 2)$
$y = 2(x+1)^2$ can be written as $y = 2(x+1)^2 + 0$ and has a turning point at $(-1, 0)$
$y = x^2 - 7$ can be written as $y = (x-0)^2 - 7$ and has a turning point at $(0, -7)$
$y = 6 - (x-2)^2$ can be written as $y = -(x-2)^2 + 6$ and has a turning point at $(2, 6)$

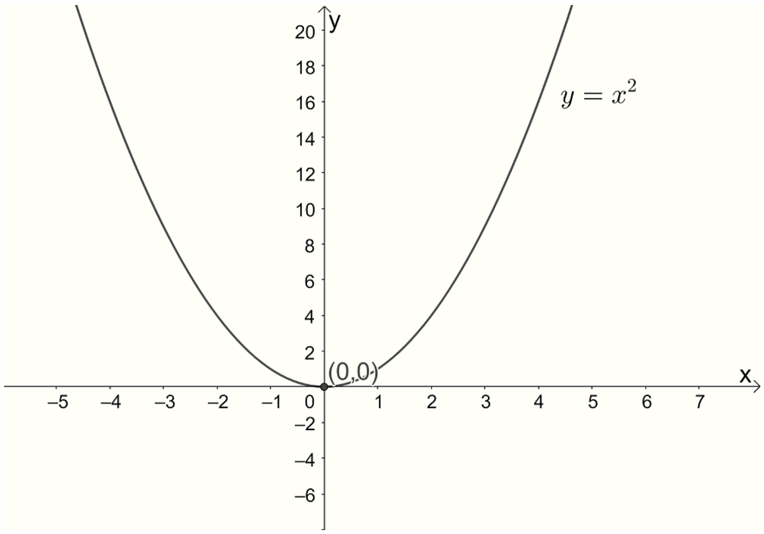
See Exercise 1

Sketching a Parabola

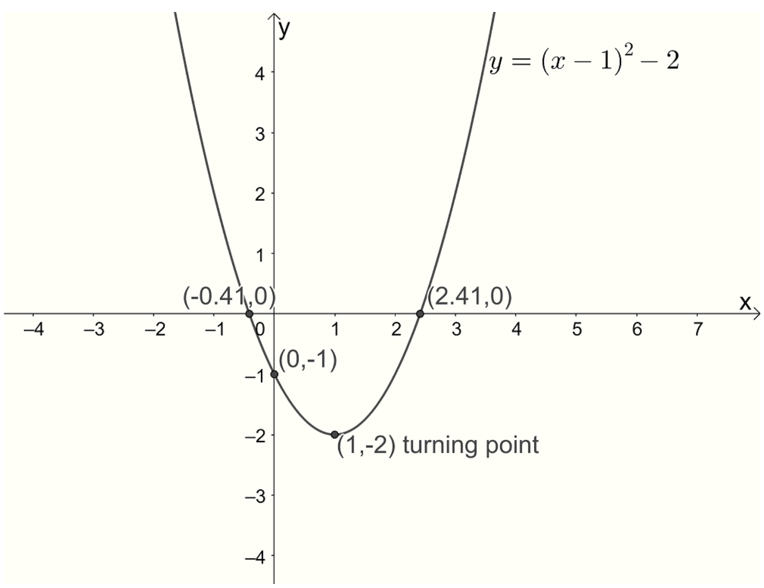
To sketch a parabola, find and label:
(a) the y-intercept (put $x = 0$)
(b) the x-intercepts (put $y = 0$)
(c) the vertex (turning point)

Examples

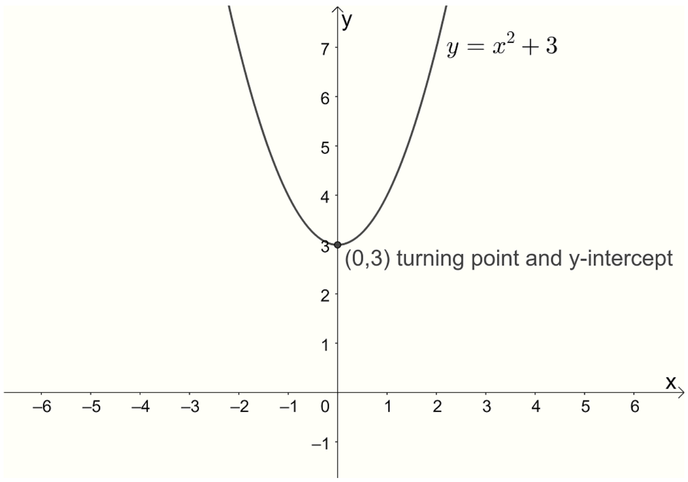
1. Sketch $y = x^2$
Intercepts $x = 0$, $y = 0$
Turning point $(0, 0)$



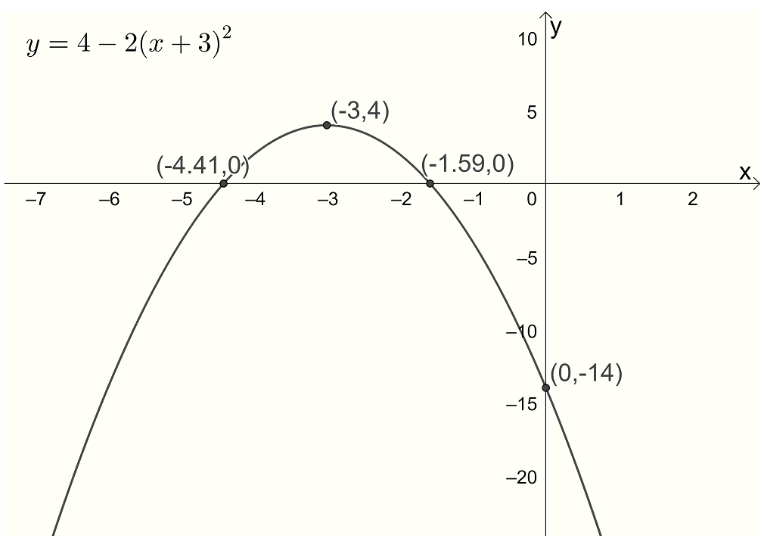
2. Sketch $y = (x-1)^2 - 2$
y-intercept: $x = 0$, $y = -1$
x-intercepts: $y = 0$, $x = \pm\sqrt{2} + 1$
Turning point: $(1, -2)$



3. Sketch $y = x^2 + 3$
y-intercept: $x = 0$, $y = 3$
x-intercepts: $y = 0$, $0 = x^2 + 3 \Rightarrow x^2 = -3$ no solution, no x-intercepts
Turning point: $(0, 3)$



4. Sketch $y = 4 - 2(x+3)^2$
y-intercept: $x = 0$, $y = -14$
x-intercepts: $y = 0$,
$0 = 4 - 2(x+3)^2$
$\Rightarrow (x+3)^2 = 2$
$\Rightarrow x + 3 = \pm\sqrt{2}$
$\Rightarrow x = -3 \pm \sqrt{2}$
Turning point: $(-3, 4)$



See Exercise 2

5. Sketch the graph $y = x^2 + 2x - 8$
y-intercept: $x = 0$, $y = -8$
x-intercepts: $y = 0$,
$0 = x^2 + 2x - 8$

$\Rightarrow 0 = (x+4)(x-2)$
$\Rightarrow x = -4$ or $x = 2$

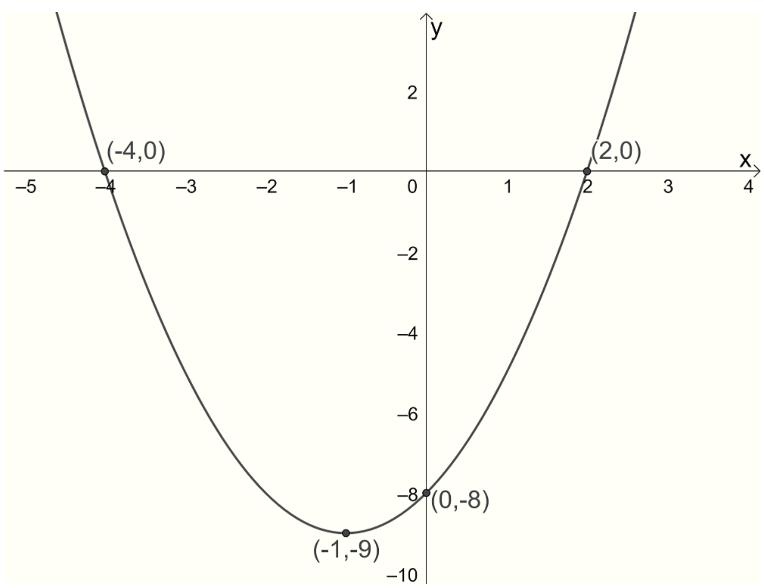
Turning point: This equation is not in turning point form so we use the equation for the x-coordinate of the turning point: $x = \left(-\frac{b}{2a}\right)$

In this example $a = 1$, $b = 2$

therefore, the x-coordinate of the turning point is $\left(-\frac{2}{2 \times 1}\right) = -1$

Since $y = x^2 + 2x - 8$ the y-coordinate of the turning point is $y = (-1)^2 + 2(-1) - 8 = -9$

$T.P. = (-1, -9)$



See Exercise 3

Exercise 1

State the turning point of the graphs of the following functions.

(a) $y = (x-1)^2 + 5$
(b) $y = 5(x-4)^2 - 12$
(c) $y = (x+2)^2 + 3$
(d) $y = -3(x+5)^2 - 3$
(e) $y = (x-6)^2$
(f) $y = -4x^2 + 3$

Answers Exercise 1

(a) $(1, 5)$ (b) $(4, 12)$ (c) $(2, 3)$ (d) $(5, 3)$ (e) $(6, 0)$ (f) $(0, 3)$

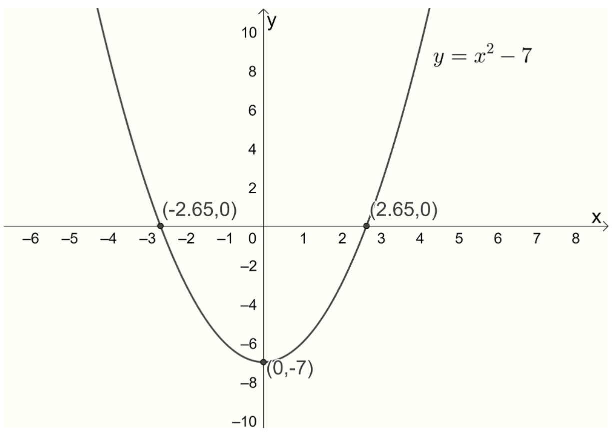
Exercise 2

Sketch graphs of the following.

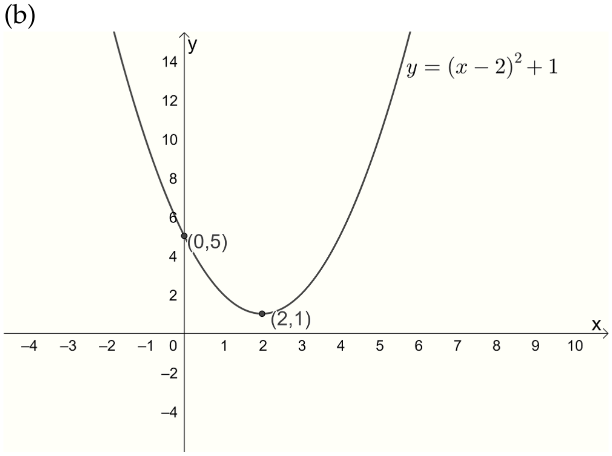
(a) $y = x^2 - 7$
(b) $y = (x-2)^2 + 1$
(c) $y = 4 - (x+3)^2$
(d) $y = (x-2)^2$
(e) $y = -(x-1)^2 - 1$

Answers Exercise 2

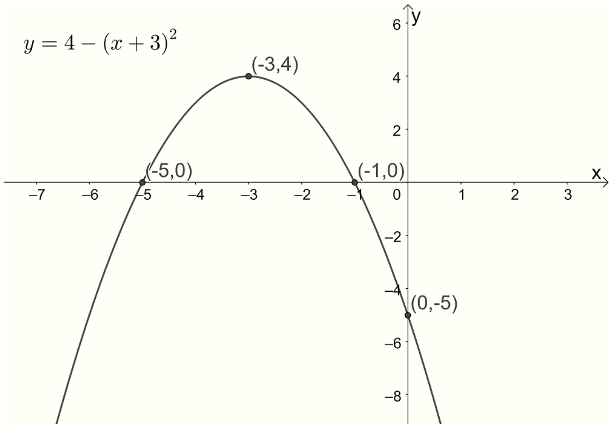
(a)



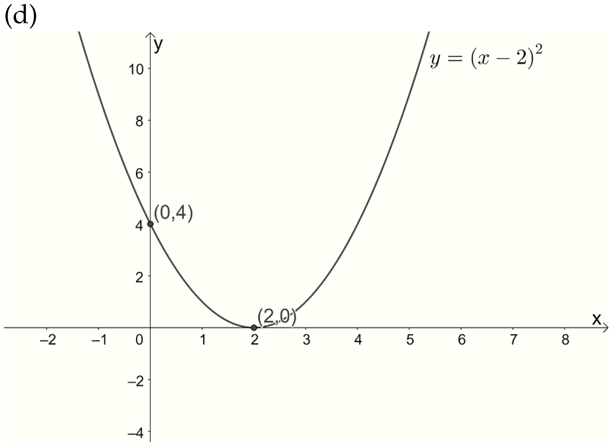
(b)



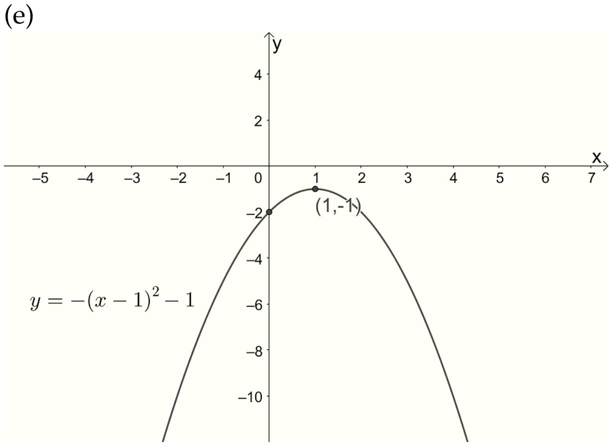
(c)



(d)



(e)



Exercise 3

Sketch the graphs of the following functions:

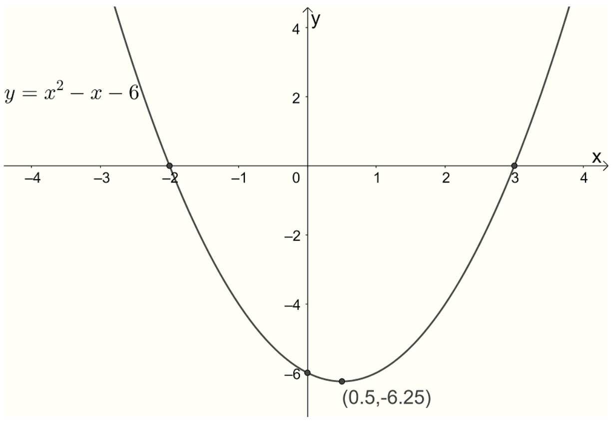
(a) $y = x^2 - x - 6$

(b) $y = -x^2 - 2x + 8$

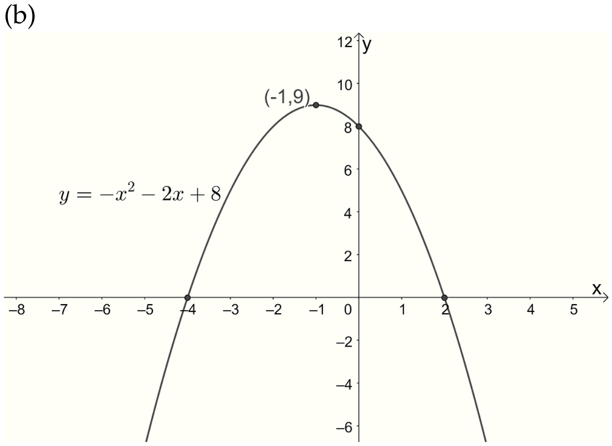
(c) $y = x^2 - 4x$
(d) $y = -2x^2 - 6x$
(e) $y = x^2 - 9$

Answers Exercise 3

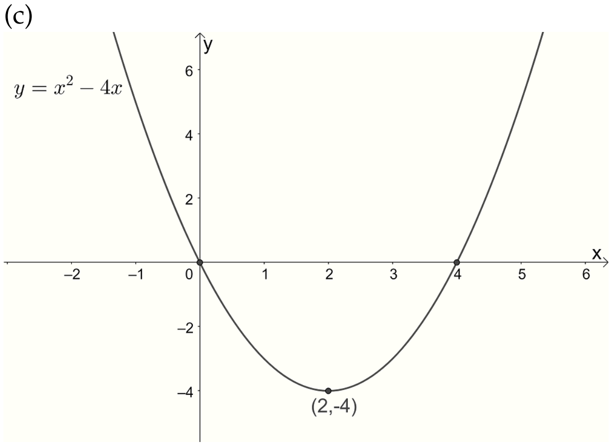
(a)



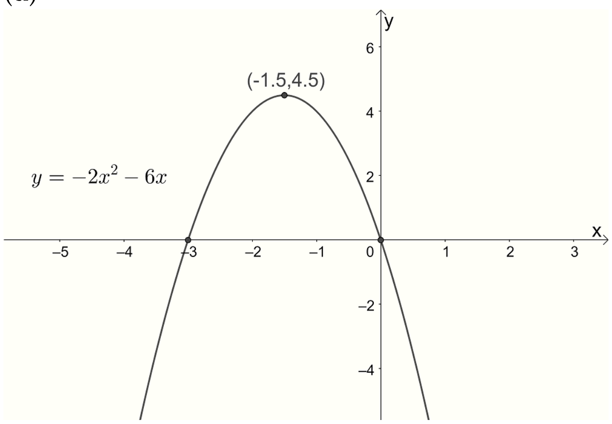
(b)



(c)

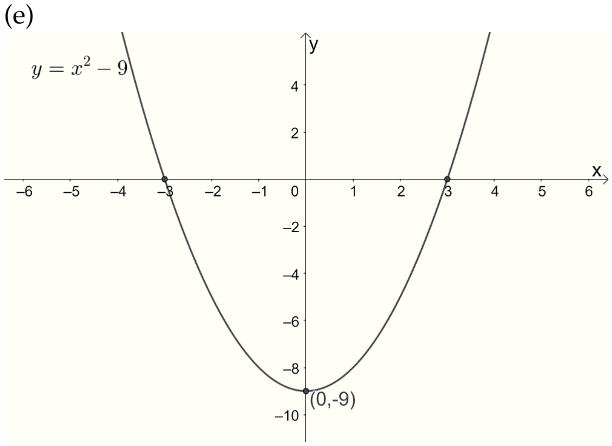


(d)



y
6
(-1.5,4.5)
4
$y = -2x^2 - 6x$
2
x
–5
–4
–3
–2
–1
0
1
2
3
–2
–4

(e)



y
$y = x^2 - 9$
4
2
x
–6
–5
–4
–3
–2
–1
0
1
2
3
4
5
6
–2
–4
–6
–8
(0,-9)
–10