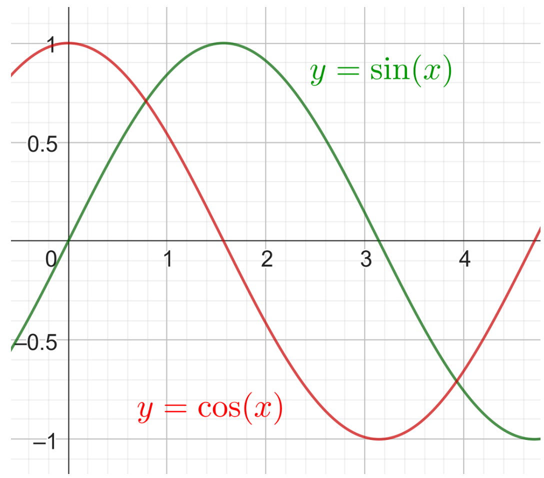
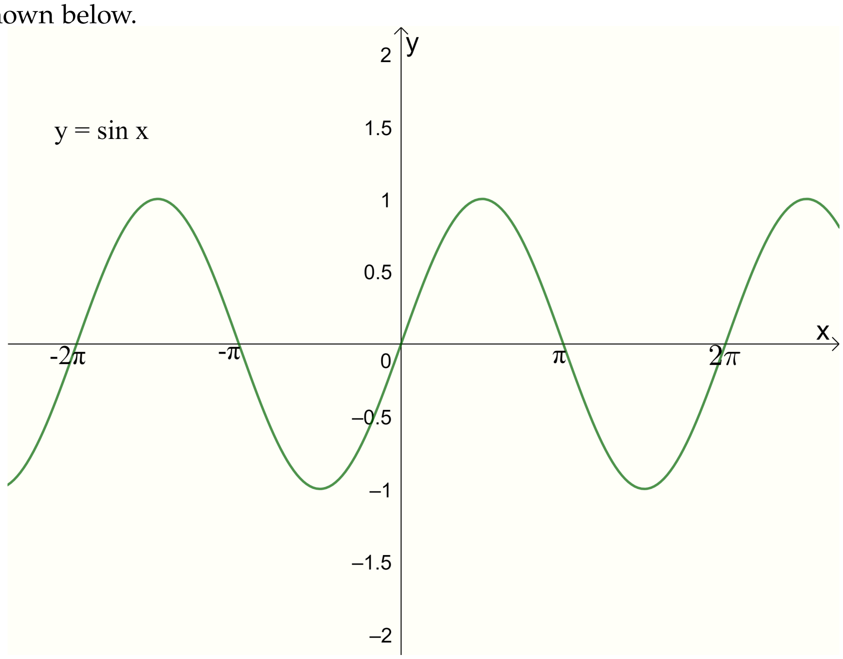
FG10: Graphs of Sine and Cosine Functions

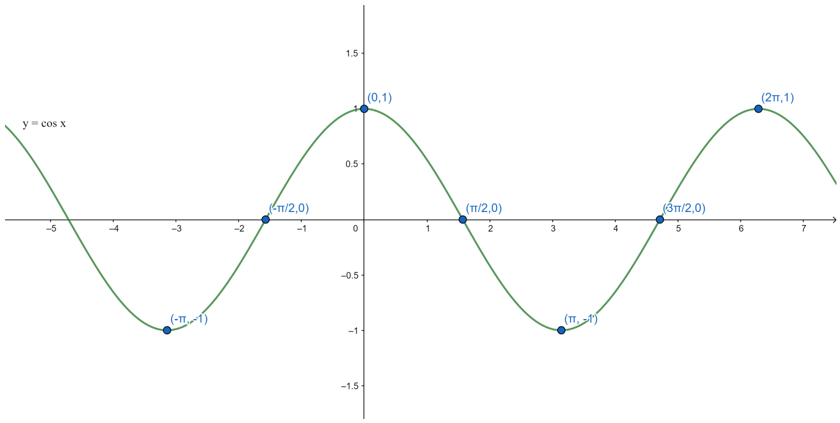


The functions $y = \sin x$ and $y = \cos x$ have a domain of $\mathbb{R}$ and a range of $[-1, 1]$.

The graphs of these functions are periodic graphs, that is, the shape of the graph repeats every set period.

The graphs of both functions have an amplitude of 1 and a period of 2π radians (that is the graph repeats every 2π units). They are shown below.





When looking at the graphs remember $\pi \approx 3.142$, so $2\pi \approx 6.284$.

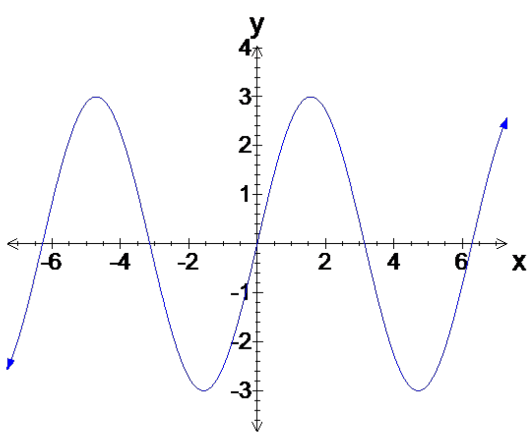
In this module we look at how the basic graphs may be transformed into graphs of more complex trigonometric functions.

Change of Amplitude and Period

The graphs of both $y = a \sin nx$ and $y = a \cos nx$ have an amplitude $|a|$ and a period of $\frac{2\pi}{n}$.

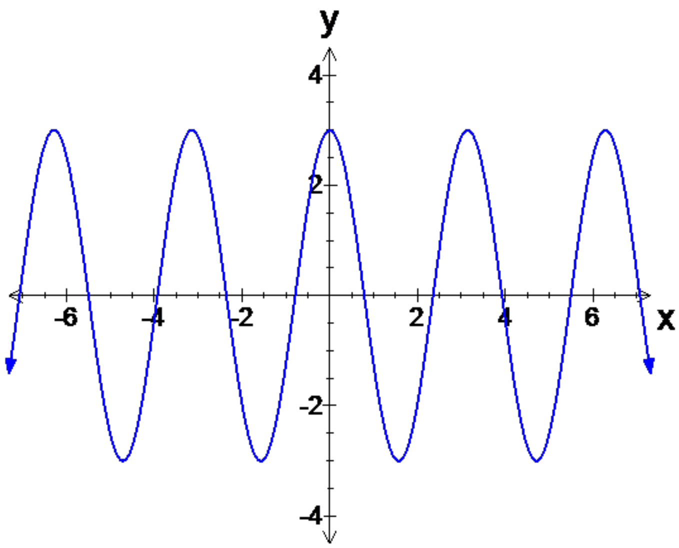
Examples

1. Graph $y = 3 \sin x$.



In this case, $a = 3$ and $n = 1$, therefore the graph has an amplitude of 3 and period of 2π.

2. Graph $y = 3 \cos 2x$.

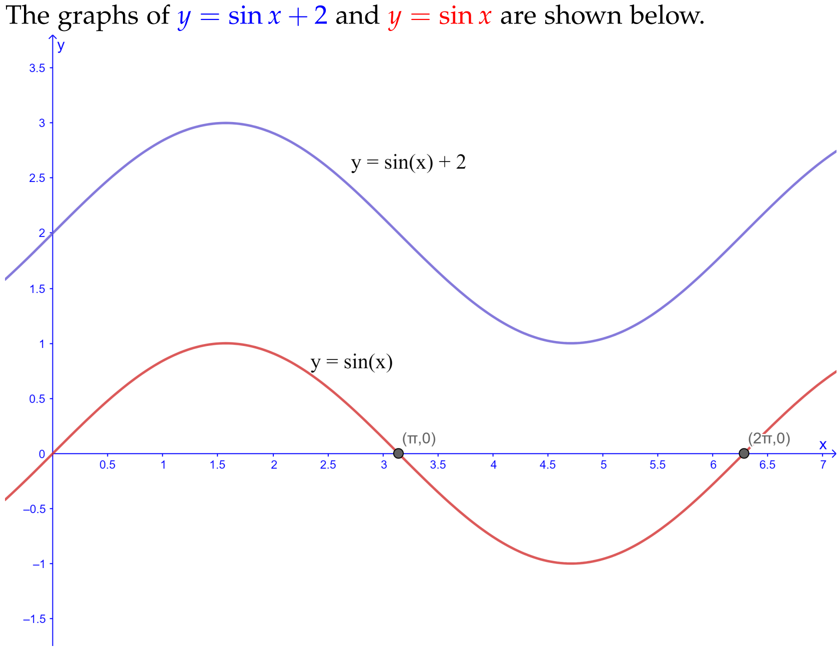


In this case, $a = 3$ and $n = 2$, therefore the graph has an amplitude of 3 and period of $\frac{2\pi}{2} = \pi$.

Vertical translation

The graph of $y = a \sin nx + k$ is the graph of $y = a \sin nx$ translated up k units (or down k units if k is negative).

The graphs of $y = \sin x + 2$ and $y = \sin x$ are shown below.



Similarly, the graph of $y = a \cos nx + k$ is the graph of $y = a \cos nx$ translated up k units (or down k units if k is negative).

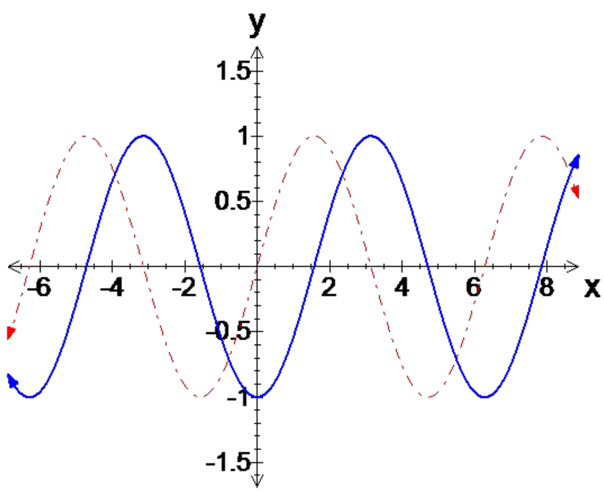
Horizontal Translation

Replacing the x with $(x - \phi)$ shifts the graphs of $y = \sin x$ and $y = \cos x$ horizontally ϕ units to the right.
Replacing the x with $(x + \phi)$ shifts the graphs of $y = \sin x$ and $y = \cos x$ horizontally ϕ units to the left.

Examples

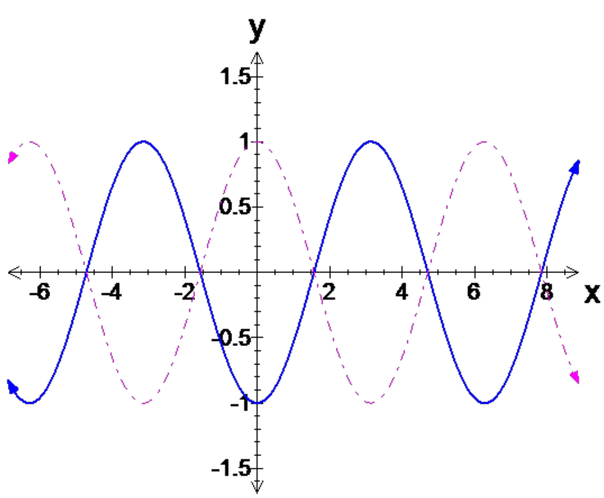
1. Graph $y = \sin\left(x - \frac{\pi}{2}\right)$

The graph of $y = \sin\left(x - \frac{\pi}{2}\right)$ shown in blue, superimposed on the graph of $y = \sin x$, in dashed red is shown below.



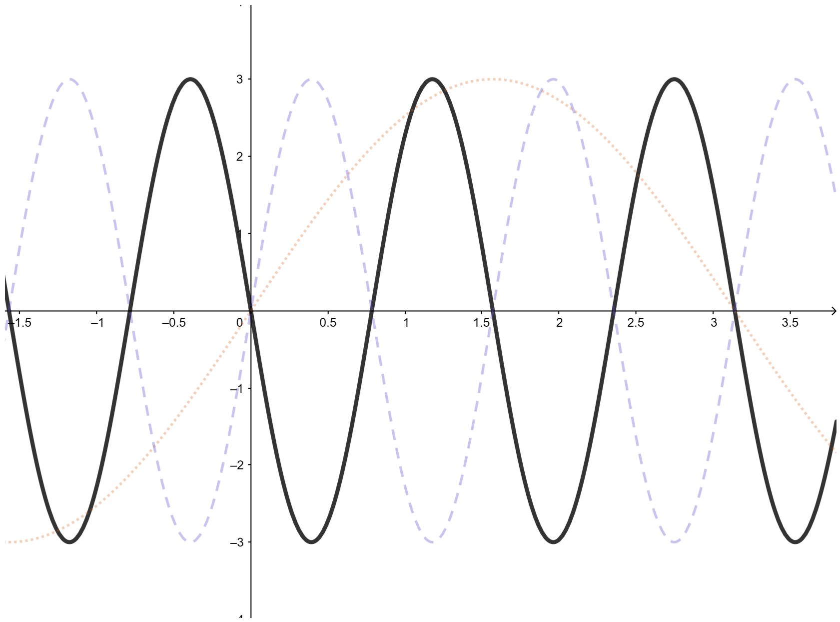
2. Graph $y = \cos(x + \pi)$

The graph of $y = \cos(x + \pi)$, shown in blue, superimposed on the graph of $y = \cos x$, in dashed red, is shown below.



3. Graph $y = 3\sin(4x - \pi)$ [1]

[1] First change $y = 3\sin(4x - \pi)$ to the form $y = 3\sin 4\left(x - \frac{\pi}{4}\right)$ so that the horizontal translation of the graph is clear.



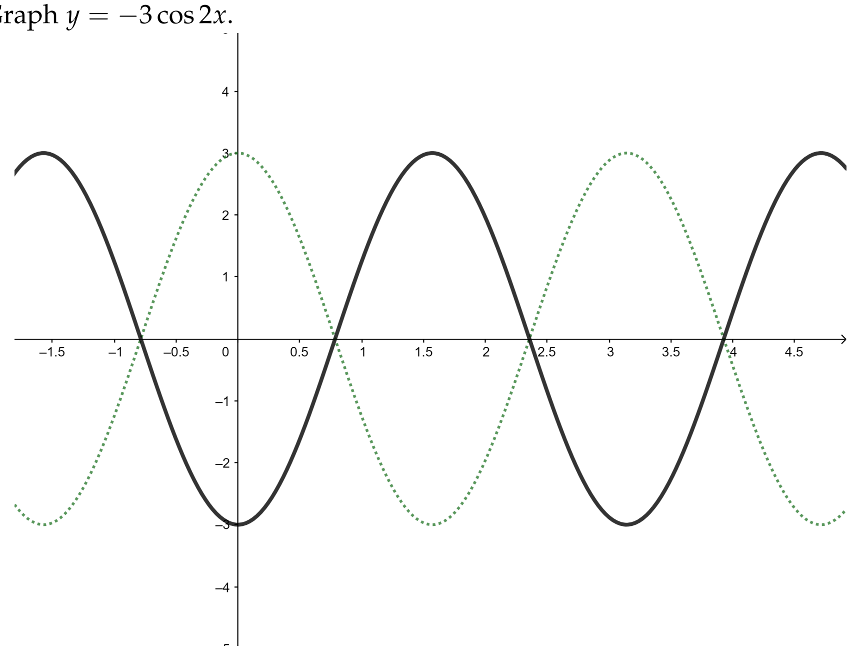
The graph of $y = 3 \sin 4 \left(x - \frac{\pi}{4}\right)$ in black is superimposed on the graphs of $y = 3 \sin x$ (dotted red) and $y = 3 \sin 4x$ (dashed grey).

Reflection

Changing the sign of a in the equations $y = a \sin nx$ and $y = a \cos nx$ results in reflection about the x-axis.

Example

Graph $y = -3 \cos 2x$.



The graph of $y = -3\cos 2x$ (in black) superimposed on the graph of $y = 3\cos 2x$ (dotted).

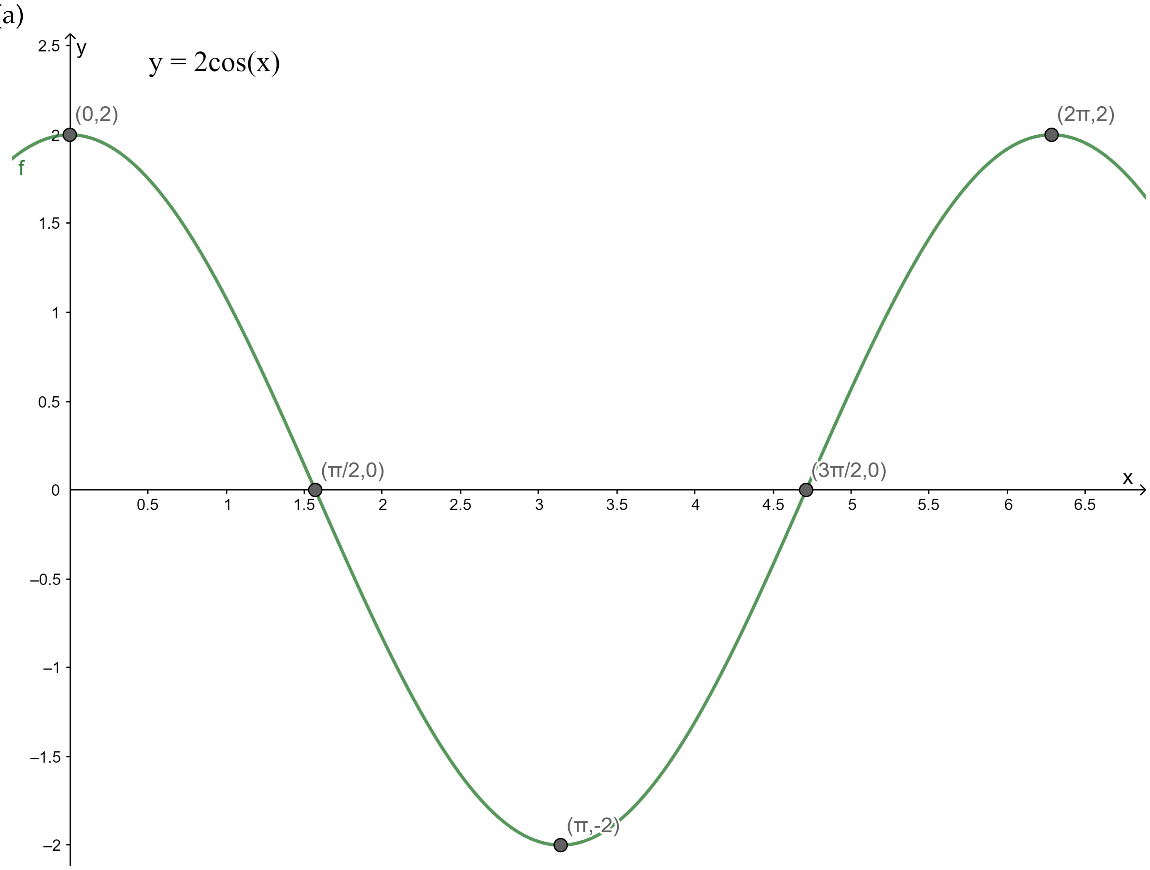
Exercise 1

1. Sketch the graphs of the following functions for one complete cycle stating the amplitude and the period.

(a) $y = 2\cos x$
(b) $y = 2\sin 3x$
(c) $y = \frac{1}{2}\sin 2x$
(d) $y = 3\cos\frac{x}{2}$
(e) $y = -2\sin 3x$

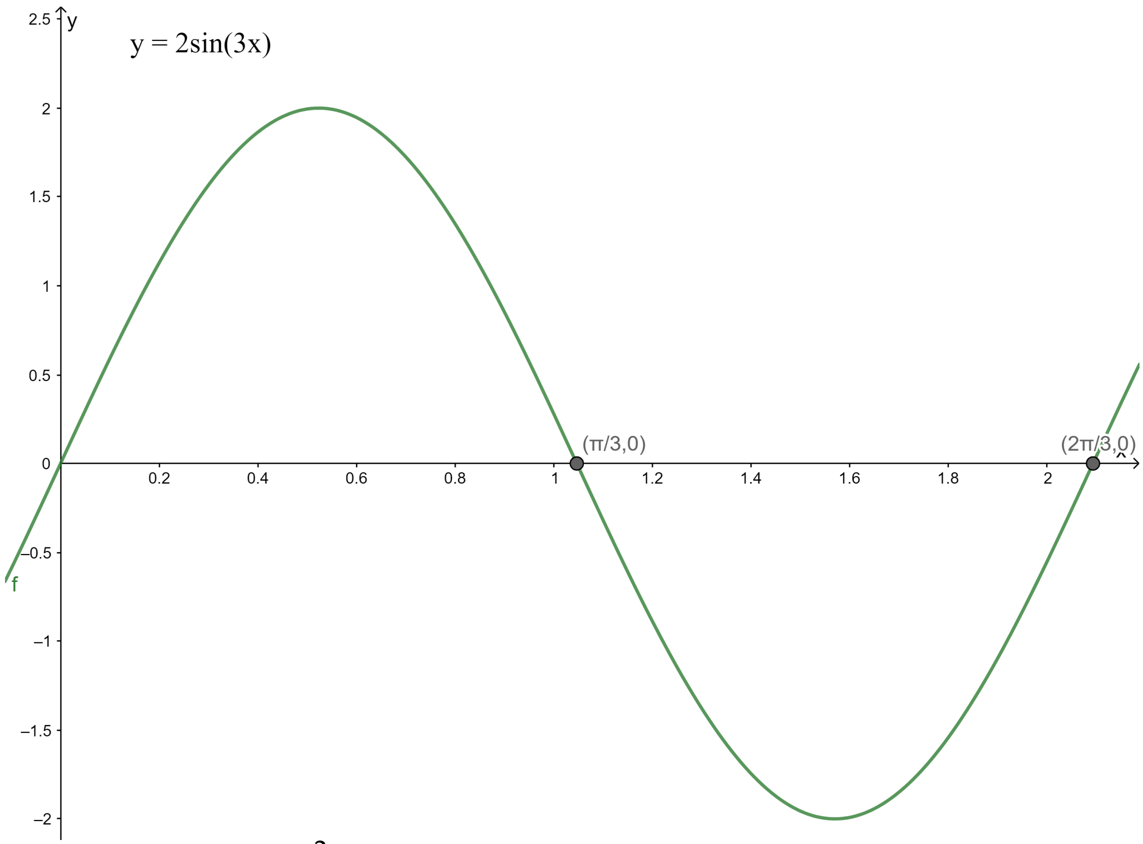
Answers

1(a)



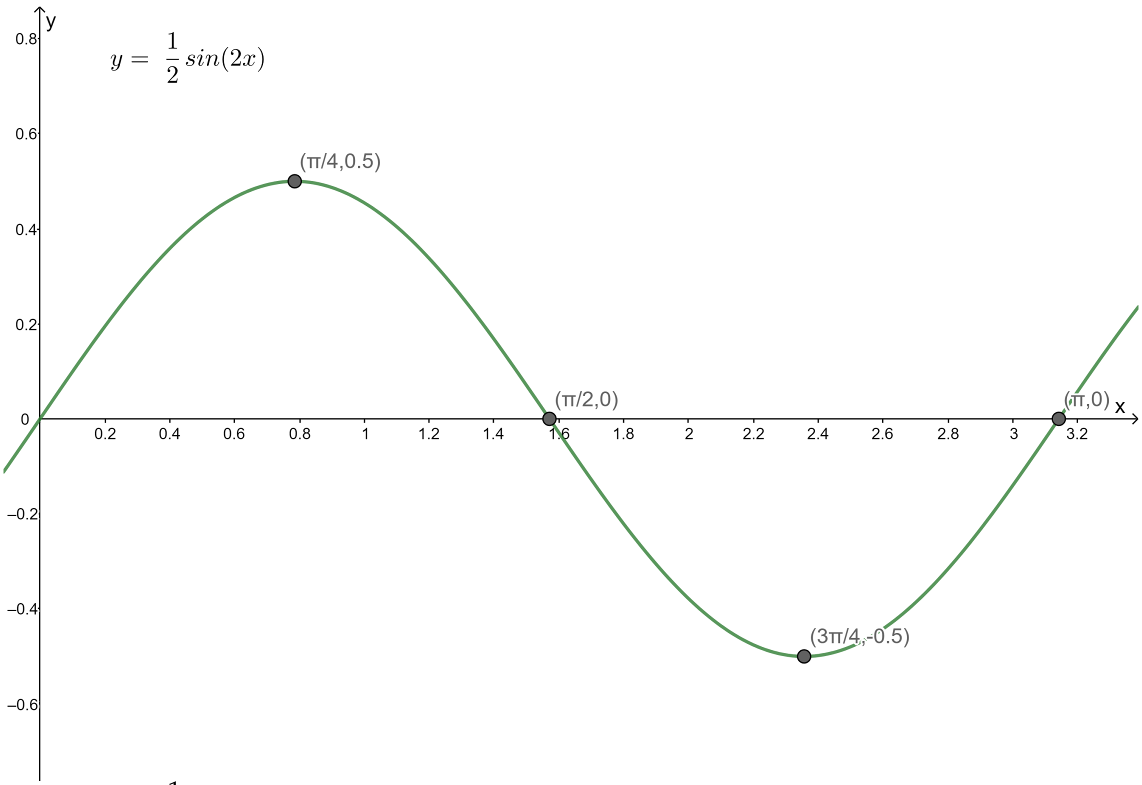
Amplitude = 2 , Period = 2π

1(b)



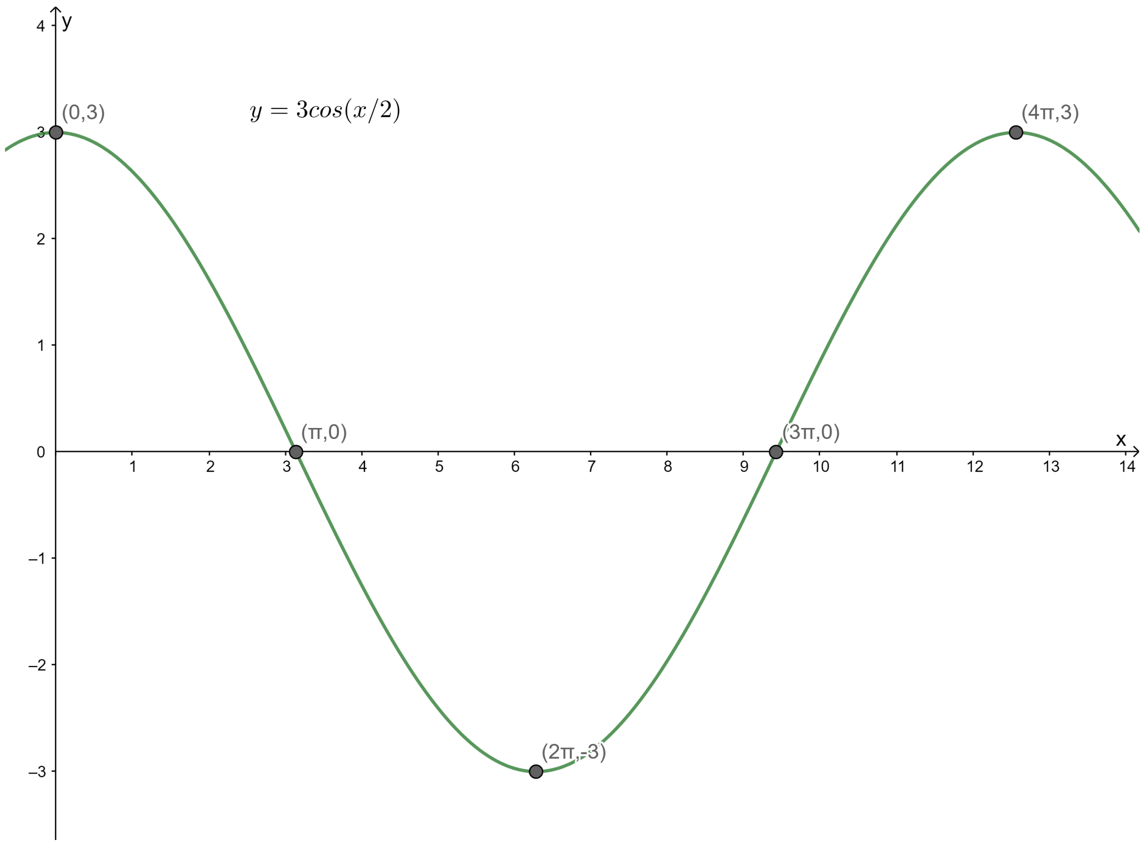
Amplitude = 2 , Period = $\frac{2\pi}{3}$

1(c)



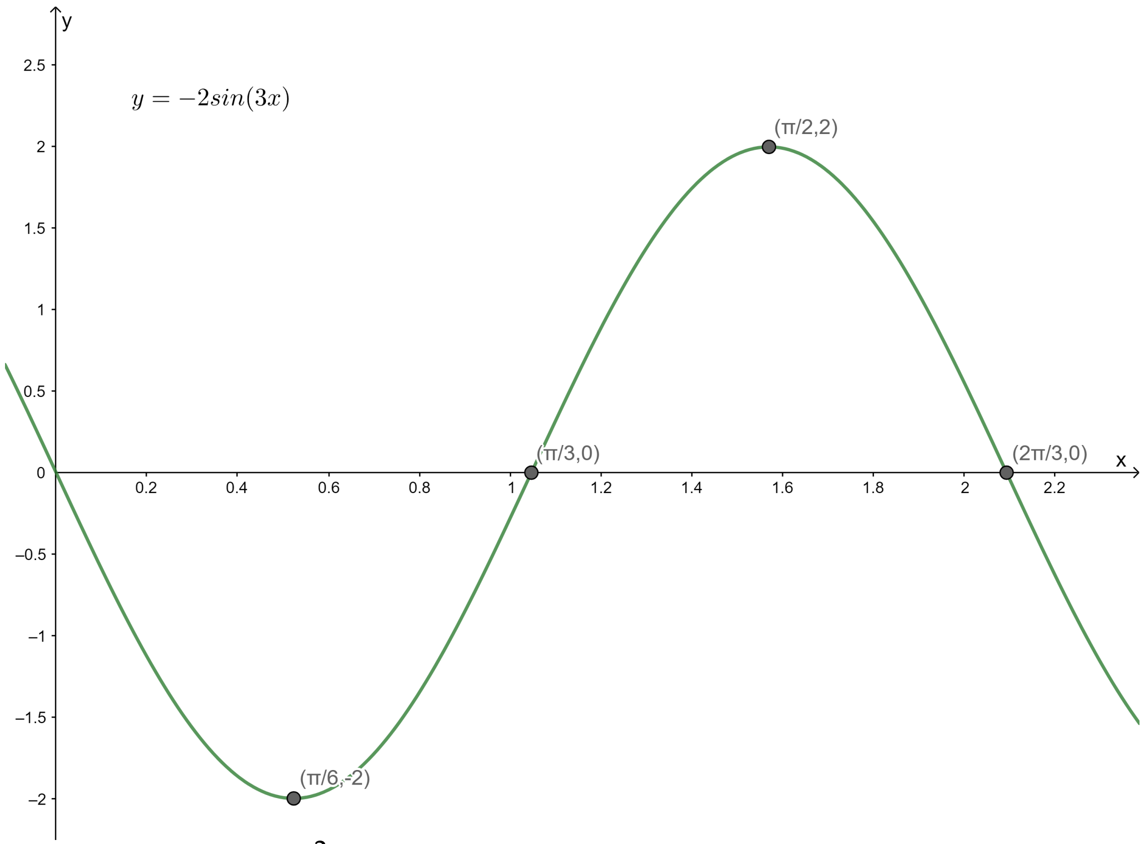
Amplitude = $\frac{1}{2}$, Period = π

1(d)



Amplitude = 3 , Period = 4π

1(e)



Amplitude = 2 , Period = $\dfrac{2\pi}{3}$

Exercise 2

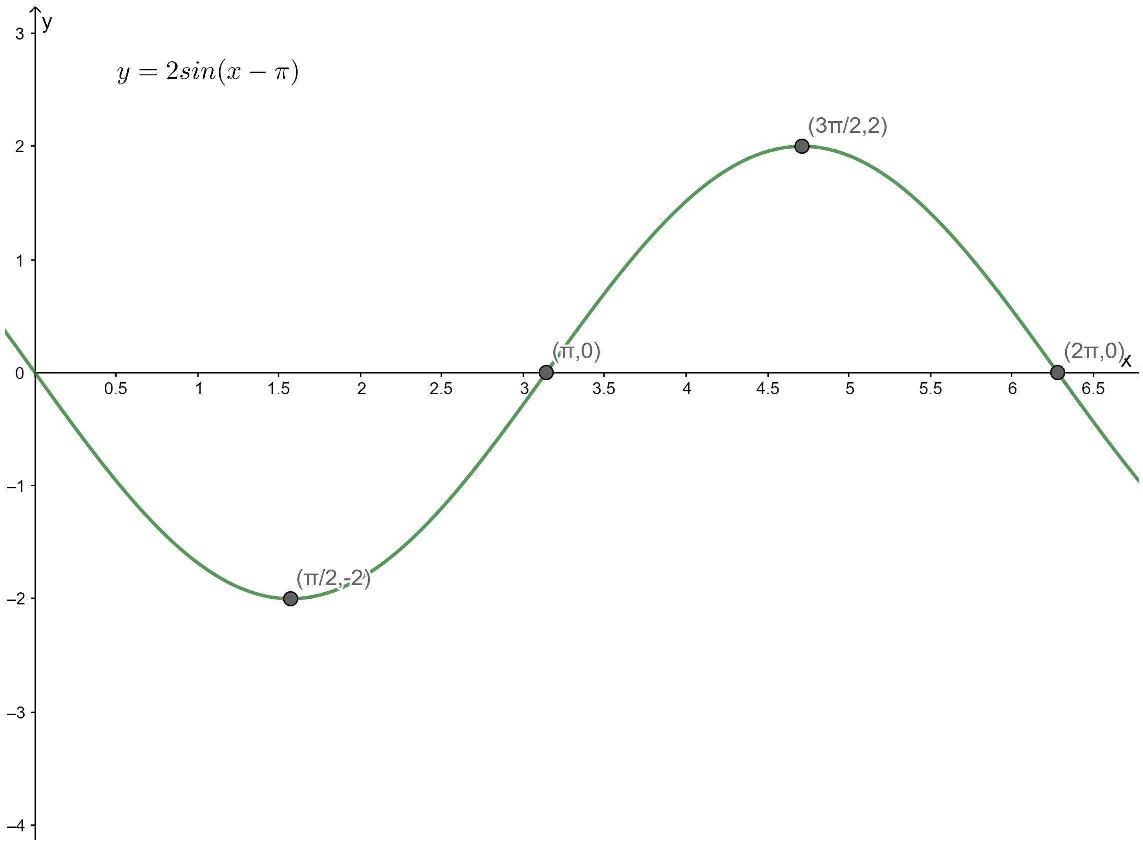
Sketch the graphs of the following functions for one complete cycle stating the amplitude and period.

(a) $y = 2\sin(x - \pi)$

(b) $y = 3\cos\left(x + \frac{\pi}{2}\right)$

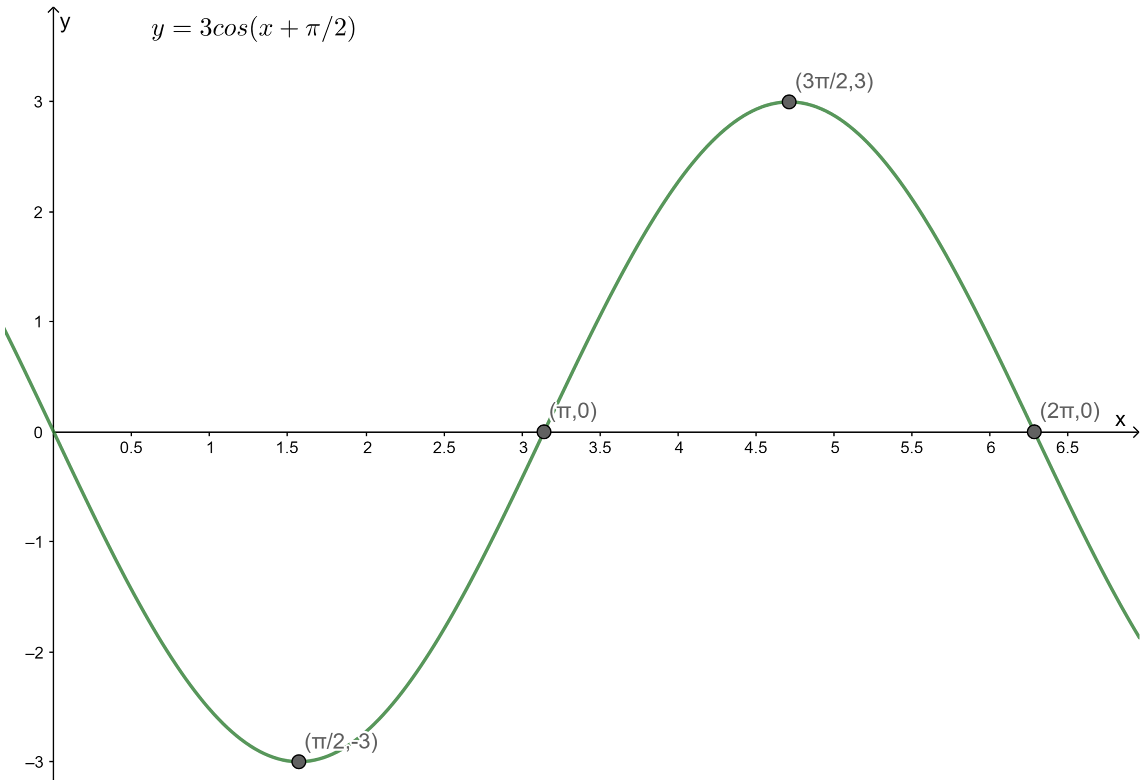
Answers

2(a)



Amplitude = 2 , Period = 2π

2(b)



Amplitude = 3 , Period = 2π

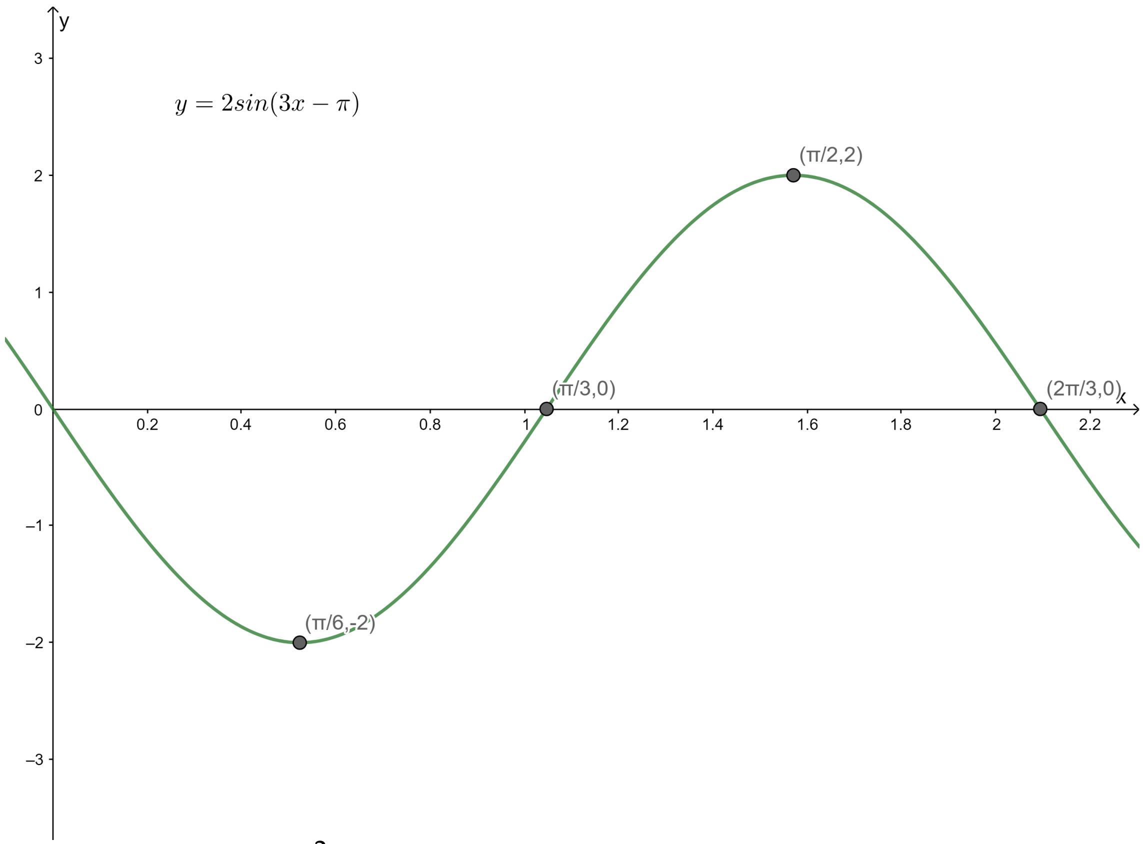
Exercise 3

Sketch the graphs of the following functions for one complete cycle stating the amplitude and period.

(a) $y = 2\sin(3x - \pi)$
(b) $y = 3\cos(4x - 2\pi)$
(c) $y = 2\sin\left(2x + \frac{\pi}{3}\right)$

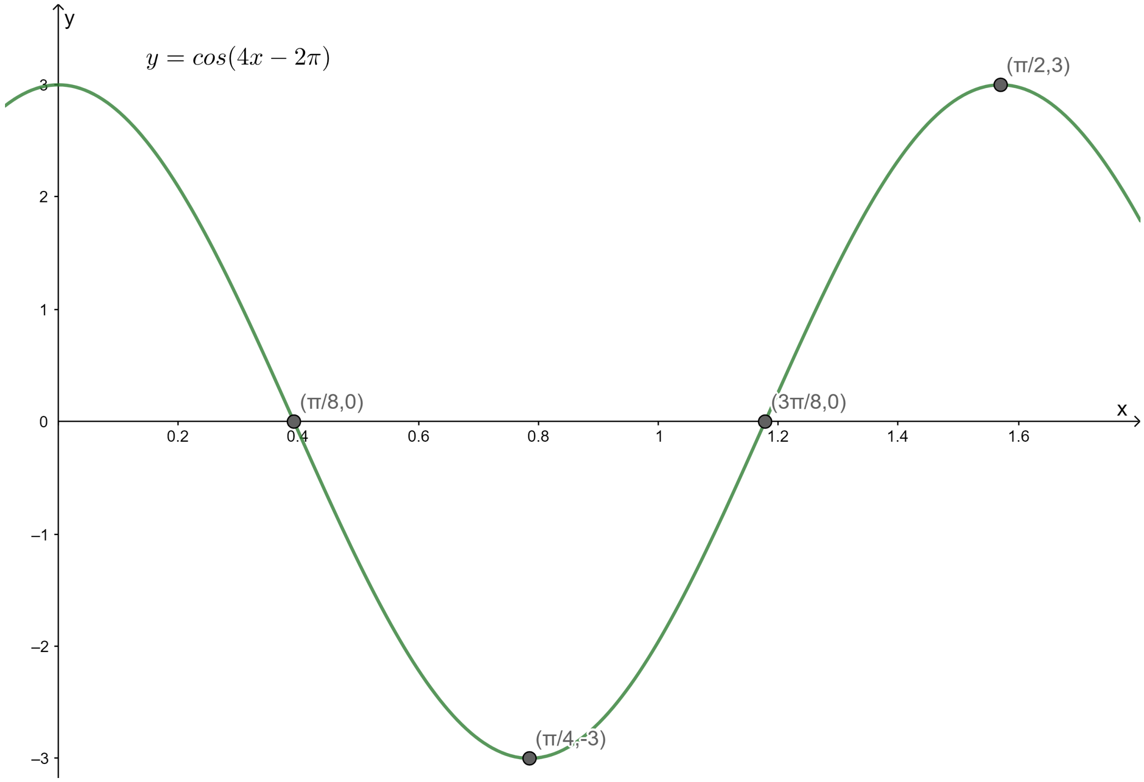
Answers

3(a)



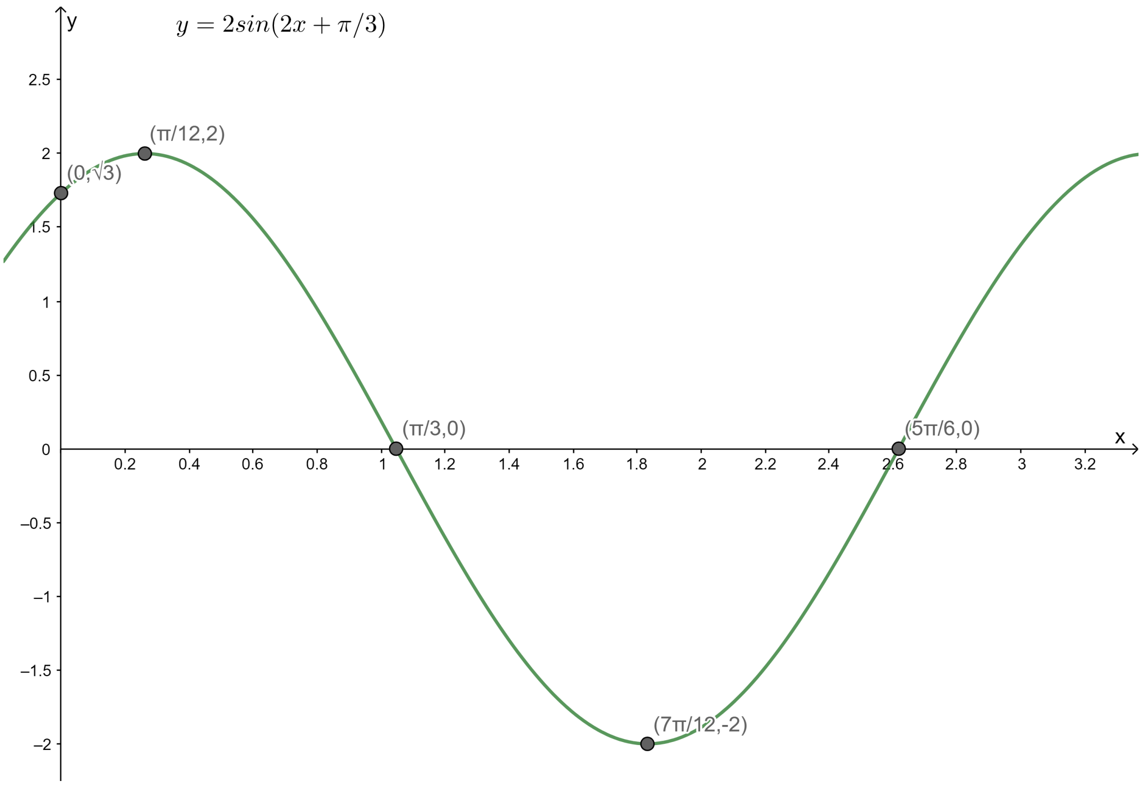
Amplitude = 2 , Period = $\frac{2\pi}{3}$

3(b)



Amplitude = 3 , Period = $\frac{\pi}{2}$

3(c)



Amplitude = 2 , Period = π