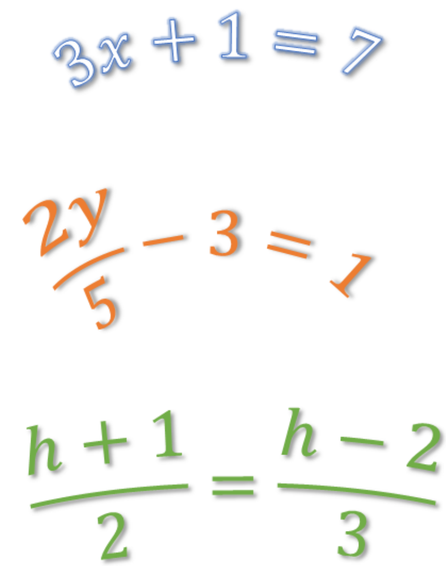
RMIT UNIVERSITY

ES1 Linear Equations



A linear equation has one unknown variable and may be solved by transposing the equation to make the variable the subject of the equation. For example, the equation

$$3\,(2x - 7) = -15$$

may be transposed to get

$$x = 6.$$

Transposition involves changing the equation via operations like addition, subtraction, multiplication and division. This module discusses techniques for solving linear equations by transposition.

Simple Equations

The essential point is that in solving an equation, what you do to one side must be done to the other side.

If you add, subtract, multiply or divide one side of the equation by a number you must do the same to the other side. This preserves equality.

Example 1

Solve $m + 4 = -2$.

Solution: We want to make m the subject[1] of the equation.

$$m + 4 = -2.$$

To get m on its own we need to take 4 from both sides[2] to get:

$$\begin{aligned} m + 4 - 4 &= -2 - 4 \\ m &= -6. \end{aligned}$$

The solution is $m = 4$. Note that you can always check your solution by substituting your answer into the original equation. In this case[3]

[1] When a variable is the only term on one side of the equal sign we call it the subject. For example the formula for the area of a circle is

$$A = \pi r^2.$$

In this case the variable A is the subject.

[2] Remember that what you do to one side of an equation must be done to the other side to preserve equality.

[3] LHS and RHS are abbreviations for left hand side and right hand side respectively.

$$\begin{aligned}
\text{LHS} &= m + 4 \\
&= -6 + 4 \\
&= -2 \\
&= \text{RHS}
\end{aligned}$$

and so our solution is correct.

Example 2

Solve $p - 2 = 5$.

Solution: We want to make p the subject. To do this we add 2 to both sides:

$$\begin{aligned}
p - 2 &= 5 \\
p - 2 + 2 &= 5 + 2 \\
p &= 7.
\end{aligned}$$

The solution is $p = 7$.

Example 3

Solve $3g = 18$.

Solution: To make g the subject we need to divide both sides by 3:

$$\begin{aligned}
3g &= 18 \\
\frac{3g}{3} &= \frac{18}{3} \\
g &= 6.
\end{aligned}$$

The solution is $g = 6$.

Example 4

Solve $y/4 = 12$.

Solution: To make y the subject we multiply both sides by 4:

$$\begin{aligned}
\frac{y}{4} &= 12 \\
\frac{y}{4} \times 4 &= 12 \times 4 \\
y &= 48.
\end{aligned}$$

The solution is $y = 48$.

More Complex Equations

Some guidelines are:

1. With more complex equations it is necessary to perform several operations. As you become more experienced, you may perform several operations in one step.
2. If the variable appears in terms on both sides of the equation, you need to get these terms to one side of the equation and then solve for the variable.
3. If an equation involves brackets, the brackets should be expanded first.
4. If an equation involves fractions, you need to use a common denominator.

These ideas are illustrated in the following examples. Note that the strategy in the examples below is not part of the solution. It is the basic plan to solve the equation. You do NOT need to provide a strategy when solving problems. Nor do you need to describe each step. The strategy and description of each step is used below to show you the thinking that leads to the solution and the steps involved.

Example 5

Solve $2w - 3 = -17$.

Strategy: We want to make w the subject. We do this in two steps. First we get $2w$ on its own then divide by two to get w as the subject. That is we add 3 to both sides and then divide both sides by 2.

Solution:

$$\begin{aligned}
2w - 3 &= -17 \\
2w - 3 + 3 &= -17 + 3 \\
2w &= -14 \\
\frac{2w}{2} &= -\frac{14}{2} \\
w &= -7.
\end{aligned}$$

The solution is $w = -7$. With more complex equations, it is a good idea to check your result. However, this is not necessary unless you are asked to check. In this example,

$$\begin{aligned}
\text{LHS} &= 2w - 3 \\
&= 2 \times (-7) - 3 \\
&= -14 - 3 \\
&= -17 \\
&= \text{RHS}
\end{aligned}$$

and so our solution is correct.

Example 6

Solve: $\frac{3d}{4} + 5 = 7$.

Strategy: First get the term involving d on its own by subtracting five from both sides, then solve for d.

Solution:

$$\begin{aligned}
\frac{3d}{4} + 5 &= 7 \\
\frac{3d}{4} + 5 - 5 &= 7 - 5 \quad \text{(subtract 5 from both sides)} \\
\frac{3d}{4} &= 2 \\
\frac{3d}{4} \times 4 &= 2 \times 4 \quad \text{(multiply both sides by 4)} \\
3d &= 8 \\
\frac{3d}{3} &= \frac{8}{3} \quad \text{(divide both sides by 3)} \\
d &= \frac{8}{3}.
\end{aligned}$$

Solution is $d = 8/3$.

We can check the solution (but this is not necessary) as follows:

$$\begin{aligned}
\text{LHS} &= \frac{3d}{4} + 5 \\
&= \frac{3}{4} \times \frac{8}{3} + 5 \\
&= 2 + 5 \\
&= 7 \\
&= \text{RHS}
\end{aligned}$$

so our solution is correct.

Example 7: Variable on Both Sides

Solve $3c + 1 = c - 5$.

Strategy: First get all the c terms on one side then solve for c.

Solution:

$$\begin{aligned}
3c + 1 &= c - 5 \\
3c - c + 1 &= c - c - 5 \quad \text{(subtract } c \text{ from both sides to get } c \text{ terms to one side)} \\
2c + 1 &= -5 \\
2c + 1 - 1 &= -5 - 1 \quad \text{(subtract 1 from both sides)} \\
2c &= -6 \\
c &= -3 \quad \text{(divide both sides by 2).}
\end{aligned}$$

The solution is $c = -3$.

Check (not required)

$$\begin{aligned} \text{LHS} &= 3c + 1 \\ &= 3\,(-3) + 1 \\ &= -8. \\ \text{RHS} &= c - 5 \\ &= -3 - 5 \\ &= -8 \\ &= \text{LHS} \end{aligned}$$

so our solution is correct.

Example 8: Brackets

Solve $3(5 - 2j) = 33$.

Strategy: First remove brackets and solve using the techniques shown above.

Solution:

$$\begin{aligned} 3(5 - 2j) &= 33 \\ 15 - 6j &= 33 \quad \text{(remove brackets)} \\ -6j &= 33 - 15 \quad \text{(subract 15 from both sides)} \\ &= 18 \\ j &= \frac{18}{-6} \quad \text{(divide both sides by } -6\text{)} \\ &= -3. \end{aligned}$$

The solution is $j = -3$.

Check (not required)

$$\begin{aligned} \text{LHS} &= 3(5 - 2j) \\ &= 15 - 6\,(-3) \\ &= 15 + 18 \\ &= 33 \\ &= \text{RHS} \end{aligned}$$

so our solution is correct.

Example 9: Brackets and Variable on Both Sides

Solve $2(3k - 2) = 5\,(k + 7)$.

Strategy: Remove brackets, get the variable on one side and then solve using the techniques shown above.

Solution:

$$\begin{aligned}
2(3k-2) &= 5\,(k+7)\\
6k-4 &= 5k+35\\
6k-5k-4 &= 35\\
k-4 &= 35\\
k &= 35+4\\
&= 39.
\end{aligned}$$

The solution is $k = 39$.

Check (not required)

$$\begin{aligned}
\text{LHS} &= 2\,(3k-2)\\
&= 6k-4\\
&= 6\times 39-4\\
&= 230.\\
\text{RHS} &= 5\,(k+7)\\
&= 5\,(39+7)\\
&= 5\times 46\\
&= 230\\
&= \text{RHS}
\end{aligned}$$

so our solution is correct.

Example 10: Fractions

Solve $\frac{h+1}{3} = \frac{h}{4}$.

Strategy: Multiply both sides of the equation by a common denominator and solve using the techniques shown above. In this case a common denominator is 12.

Solution:

$$\begin{aligned}
\frac{h+1}{3} &= \frac{h}{4}\\
\left(\frac{h+1}{3}\right)\times 12 &= \frac{h}{4}\times 12\\
4\,(h+1) &= 3h\\
4h-3h+4 &= 0\\
h+4 &= 0\\
h &= -4.
\end{aligned}$$

The solution is $h = -4$.

Check (note required)

$$\begin{aligned} \text{LHS} &= \frac{h+1}{3} \\ &= \frac{-4+1}{3} \\ &= -1 \\ \text{RHS} &= \frac{h}{4} \\ &= -1 \\ &= \text{LHS} \end{aligned}$$

so our solution is correct.

Example 11

Solve $\frac{2z+11}{7} = \frac{z-3}{12}$.

Strategy: A common denominator is 84. So we multiply both sides by 84 and solve using the techniques shown above.

Solution:

$$\begin{aligned} \frac{2z+11}{7} &= \frac{z-3}{12} \\ \left(\frac{2z+11}{7}\right) \times 84 &= \left(\frac{z-3}{12}\right) \times 84 \\ 12\,(2z+11) &= 7\,(z-3) \\ 24z + 132 &= 7z - 21 \\ 24z - 7z + 132 &= -21 \\ 17z + 132 &= -21 \\ 17z &= -21 - 132 \\ &= -153 \\ z &= \frac{-153}{17} \\ &= -9. \end{aligned}$$

The solution is $z = -9$.

Note that in this example, we have not put in all the details and explanation. This is what you should aim to do.

Example 12

Solve $\frac{3u}{4} - \frac{1}{3} = 7$.

Strategy: A common denominator is 12. So we multiply both sides by 12 and solve using the techniques shown above.

Solution:

$$\begin{aligned}
\frac{3u}{4} - \frac{1}{3} &= 7 \\
\left(\frac{3u}{4} - \frac{1}{3}\right) \times 12 &= 7 \times 12 \\
\frac{3u}{4} \times 12 - \frac{1}{3} \times 12 &= 84 \\
9u - 4 &= 84 \\
9u &= 88 \\
u &= \frac{88}{9}.
\end{aligned}$$

The last example is important as it shows that the solution may not be a whole number as in the previous examples.

Exercise 1

Solve the following equations.

a) $x + 3 = 7$	*b*) $5 - j = -2$	*c*) $3c = 12$
d) $-r = -12$	*e*) $\frac{m}{2} = -7$	*f*) $-8u = 12$
g) $4g + 4 = 16$	*h*) $7 - 2w = 1$	*i*) $\frac{e}{2} - 5 = -8$
j) $21 - 3t = 12$	*k*) $\frac{y}{5} - 9 = -5$	*l*) $3 - \frac{u}{2} = -7$

Answers

1. *a*) 4 *b*) 7 *c*) 4 *d*) 12 *e*) -14 *f*) $-\frac{3}{2}$ *g*) 3 *h*) 3
   i) -6 *j*) 3 *k*) 20 *l*) 20

Exercise 2

Solve these equations.

a) $5i + 2 = i + 10$	*b*) $10p - 11 = 2p - 3$	*c*) $5a - 12 = 3a + 6$
d) $10d + 10 = 0$	*e*) $f + 6 = 6f - 9$	*f*) $8 - g = 5g + 14$
g) $5h - 2 = 7h - 12$	*h*) $6j + 13 = 4j + 13$	

Answers

2. *a*) 2 *b*) 1 *c*) 9 *d*) -1 *e*) 3 *f*) -1 *g*) 5 *h*) 0

Exercise 3

Solve

a) $3(2k-4)=18$ *b*) $5(2z+9)=15$ *c*) $3(x+4)=6$

d) $3(c+3)+2(c-5)=4$ *e*) $3(2v-3)+2(v-4)=-25$ *f*) $3(b+4)=2(4b+1)$

Answers

3. *a*) 5 *b*) −3 *c*) −2 *d*) 1 *e*) −1 *f*) 2

Exercise 4

Solve

a) $\frac{9n}{5}-4=5$ *b*) $\frac{4m}{3}-11=9$ *c*) $1-\frac{9q}{2}=-8$

d) $\frac{w-4}{2}=2$ *e*) $\frac{3-2e}{11}=1$ *f*) $\frac{3r+9}{5}=-3$

g) $\frac{3t}{8}+4=1$ *h*) $\frac{y}{3}=\frac{2}{9}$ *i*) $\frac{5u-4}{4}=\frac{u-5}{5}$

j) $\frac{2i+1}{7}=\frac{3i-5}{4}$ *k*) $\frac{p+1}{3}+1=4$ *l*) $2-\frac{5a-4}{4}=4$

m) $\frac{d-3}{3}-4=\frac{d-2}{2}$ *n*) $\frac{1-m}{5}-m=\frac{2m-1}{2}$

Answers

4. *a*) 5 *b*) 15 *c*) 2 *d*) 8 *e*) −4 *f*) −8 *g*) −8 *h*) $\frac{2}{3}$
i) 0 *j*) 3 *k*) 8 *l*) $-\frac{4}{5}$ *m*) −24 *n*) $\frac{7}{22}$