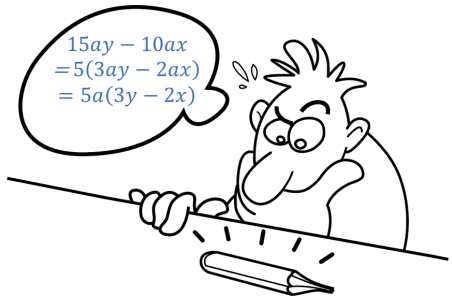
# A3.1 Factorisation: Common Factors

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This module looks at using common factors to factorise mathematical expressions.

Watch a short video on factorisation using common factors

Click to download a transcription of the video on Factorisation using common factors

## *Expansion*

Expansion of brackets (or removing brackets) in an algebraic expression is done by multiplying all the terms inside the brackets by the term(s) outside the brackets.

Examples:

$$\begin{aligned} 6(7) &= 42 \\ 6(b+1) &= 6b+6 \\ 3(4x+5) &= 12x+15 \\ 5a(3y-2x) &= 15ay-10ax \end{aligned}$$

## *Factorisation is the Reverse of Expansion.*

Factors are numbers we can multiply together to get another number.

To factorise a number or algebraic expression means to write the number or expression as a product (multiplication) of numbers or expressions.

Examples:

1. $42 = 6 \times 7 = 2 \times 3 \times 7$ has factors 6 and 7 but also has factors of 2 and 3 .
2. $-2xyz = -1 \times 2 \times x \times y \times z$ has factors $-1$ , 2 , $x$ , $y$ , $z$ but remember that combinations are factors such as $-2 \times xy \times z$ which means that $-2$ , $xy$, $xyz$, and $z$ are factors as well.
3. $6b+6 = 6(b+1) = 2 \times 3 \times (b+1)$ has factors 6 and $(b+1)$ as well as 2 and 3, the Highest Common Factor of $6b+6$ is 6 .

4. $12x + 15 = 3(4x + 5)$ has factors 3 and $4x + 5$. The Highest Common Factor of $(12x + 15)$ is 3.

5. $15ay - 10ax = 5a(3y - 2x)$ has factors 5 , $a$ and $(3y - 2x)$ , $5a$ and $5\,(3y - 2x)$. The Highest Common Factor of $(3y - 2x)$ is $5a$.

6. $4prs + 16pr + 2ps + 8p = 2p(2rs + 8r + s + 4) = 2p(2r + 1)(s + 4)$ has factors 2 , $p$ , $(2r + 1)$ , $(s + 4)$ as well as $(2rx + 8r + s + 4)$ as well as $2p$ .

| |
|---|
| Expansion means removing brackets<br>Factorisation means inserting brackets |

## *Factorisation by Removing a Common Factor*

The steps are:

- Search each term in the expression for a common factor (every term must have this factor)
- There may be several common factors. Search until you have found all of them
- If there is more than one common factor multiply them to give Highest Common Factor . (HCF)
- The HCF is placed before the bracket.
- The terms inside the bracket are found by dividing each term by the HCF.

*Examples:*

1.

$$\begin{aligned} 5y + 10 &= 5 \times y + 5 \times 2 \\ &= 5(y + 2) \\ &\quad common\ factor\ of\ 5 \end{aligned}$$

2.

$$\begin{aligned} 3x + 3y &= 3 \times x + 3 \times y \\ &= 3(x + y) \\ &\quad common\ factor\ of\ 3 \end{aligned}$$

3.

$$\begin{aligned} p^2 + p &= p \times p + p \times 1 \\ &= p(p + 1) \\ &\quad common\ factor\ of\ p \end{aligned}$$

4.

$$\begin{aligned} 7y^2 + 7y &= 7y \times y + 7y \times 1 \\ &= 7y(y + 1) \\ &\quad common\ factors\ of\ 7\ and\ y \\ &\quad HCF = 7y \end{aligned}$$

5.

$$\begin{aligned} 2abc - 12ac &= 2a \times bc - 2a \times 6c \\ &= 2ac \times b - 2ac \times 6 \\ &= 2ac(b - 6) \\ &\quad common\ factors\ of\ 2,\ a,\ and\ c \\ &\quad HCF = 2ac \end{aligned}$$

**See exercise 1.**

*Further Examples (Negative Factors)*

6.

$$\begin{aligned} -2a - 2b &= (-2) \times a + (-2) \times b \\ &= -2(a + b) \\ &\quad common\ factor\ of\ -2 \end{aligned}$$

7.

$$\begin{aligned} -3x + 6xy &= (-3x) \times 1 - (-3x) \times 2y \\ &= -3x(1 - 2y) && HCF = -3x \\ &= 3x(-1 + 2y) && HCF = 3x \\ &= 3x(2y - 1) \end{aligned}$$

See exercise 2

## *Exercise 1*

Factorise the following expressions (if possible):

| | | | | | |
|---|---|---|---|---|---|
| a) | $3x + 3y$ | b) | $2a - 2b$ | c) | $8a - 8b + 8c$ |
| d) | $xy - 5x$ | e) | $x^2 - x$ | f) | $7x + 21y$ |
| g) | $5m - 2n$ | h) | $c^2 - 2bc - 3c$ | i) | $5mn - 10n$ |
| j) | $3m^2 - 3mnp$ | k) | $7x + 21x^2$ | l) | $12m^2 - 18mn$ |
| m) | $5xy - 10xz$ | n) | $5pq - pq^2 - 3pqr$ | o) | $2ab^2c + 6abc^2$ |
| p) | $rst + 5rst^2 - 2rs$ | q) | $5mn + 10m - pqr$ | r) | $5xyz - x^2yz^2 + 10x$ |

## *Answers Exercise 1*

| | | | | | |
|---|---|---|---|---|---|
| a) | $3(x + y)$ | b) | $2(a - b)$ | c) | $8(a - b + c)$ |
| d) | $x(y - 5)$ | e) | $x(x - 1)$ | f) | $7(x + 3y)$ |
| g) | $no\ factors$ | h) | $c(c - 2b - 3)$ | i) | $5n(m - 2)$ |
| j) | $3m(m - np)$ | k) | $7x(1 + 3x)$ | l) | $6m(2m - 3n)$ |
| m) | $rs(t + 5t^2 - 2)$ | n) | $pq(5 - q - 3r)$ | o) | $2abc(b + 3c)$ |
| p) | $rs\left(t + 5t^2 - 2\right)$ | q) | $no\ factors$ | r) | $x(5yz - xyz^2 + 10)$ |

## *Exercise 2*

Factorise the following by removing a negative factor.

| | | | | | |
|---|---|---|---|---|---|
| a) | $-3x - 6y$ | b) | $-15xy + 25xz$ | c) | $-2xy + 4xyz$ |
| d) | $14xyz - 7xy$ | e) | $-6xyz - 15yz - 3xy^2z$ | f) | $7x - 21y$ |

## *Answers Exercise 2*

| | | | | | |
|---|---|---|---|---|---|
| a) | $-3(x + 2y)$ | b) | $-5x(y - 5z)$ | c) | $-2xy(1 - 2z)$ |
| d) | $7xy(2z - 1)$ | e) | $-3yz(2x + 5 + xy)$ | f) | $7(x - 3y)$ |