A1.3 Removing Brackets

$$a(b+c) = ab + ac$$

Brackets are commonly used to express mathematical formulae. In order to manipulate expressions containing brackets it is necessary to remove them first. This module discusses how to remove brackets.

Play a short video on removing brackets.

Get a transcript of the video.

The Distributive Law

In what follow we will write $a\,(b+c)$ which means $a \times (b+c)$. That is the a multiplies the bracket. We usually don't include the times sign $(\times)$.

The distributive law says that

$$a\,(b+c) = ab + ac.$$

In words, the a outside the bracket multiplies everything inside the bracket.

For example,

$$2\,(3+5) = 2 \times 3 + 2 \times 5.$$

Removing the brackets is called expansion.

Examples

- Expand $3\,(x+2)$. We have

$$\begin{aligned} 3\,(x+2) &= 3 \times x + 3 \times 2 \\ &= 3x + 6. \end{aligned}$$

- Expand $-5\,(m+4)$. We have

$$\begin{aligned} -5\,(m+4) &= (-5) \times m + (-5) \times 4 \\ &= -5m - 20. \end{aligned}$$

Usually we omit the first step shown above and expand directly as shown in the following examples.

- Expand $-2(p-7)$. We have

$$-2(p-7) = -2p + 14.$$

- Expand $e(e+2)$. We have

$$e(e+2) = e^2 + 2e.$$

- Expand $-(4p-3)$. We have

$$-(4p-3) = -4p + 3.$$

- Expand and simplify $4(x-3) + 2(x+1)$. We have

$$\begin{aligned} 4(x-3) + 2(x+1) &= 4x - 12 + 2x + 2 \\ &= 6x - 10. \end{aligned}$$

- Expand and simplify $6k(2k+3) - 3k(k-3)$. We have

$$\begin{aligned} 6k(2k+3) - 3k(k-3) &= 12k^2 + 18k - 3k^2 + 9k \\ &= 9k^2 + 27k. \end{aligned}$$

Now do some practice. See Exercise 1 below.

Binomial Products

A binomial product is something like $(m+2)(m+3)$. It is the product of two binomial terms[1].

To expand the two brackets, multiply each term in the first bracket by each term in the second bracket. Then simplify if possible. For example:

$$\begin{aligned} (m+2)(m+3) &= m(m+3) + 2(m+3) \\ &= m^2 + 3m + 2m + 6 \\ &= m^2 + 5m + 6. \end{aligned}$$

Symbolically we can write:

$$(a+b)(c+d) = ac + ad + bc + bd.$$

[1] A binomial term is an expression involving two terms. Examples are: $a+3$, $1+c$, $a+b$, and $(3x+2y)$.

Two Important Cases

Special cases arise when the binomial terms are the same. That is $(a+b)(a+b) = (a+b)^2$ or $(a-b)(a-b) = (a-b)^2$.

We have

$$\begin{aligned}(a+b)^2 &= (a+b)(a+b)\\ &= a^2+ab+ba+b^2\\ &= a^2+2ab+b^2 \qquad (1)\end{aligned}$$

and

$$\begin{aligned}(a-b)^2 &= (a-b)(a-b)\\ &= a^2-ab-ba+b^2\\ &= a^2-2ab+b^2. \qquad (2)\end{aligned}$$

These occur often in mathematics and you should commit them to memory.

Examples

- Expand $(2f+3)(g+5h)$. We have

$$\begin{aligned}(2f+3)(g+5h) &= 2fg+(2f)(5h)+3g+15h\\ &= 2fg+10fh+3g+15h.\end{aligned}$$

  In this case not further simplification is possible.

- Expand $(p-2)(p-7)$. We have

$$\begin{aligned}(p-2)(p-7) &= p^2-7p-2p+14\\ &= p^2-9p+14.\end{aligned}$$

- Expand $(2w+3)(w-4)$. We have

$$\begin{aligned}(2w+3)(w-4) &= 2w^2-8w+3w-12\\ &= 2w^2-5w-12.\end{aligned}$$

- Expand $(4r-3s)^2$. We have

$$\begin{aligned}(4r-3s)^2 &= (4r-3s)(4r-3s)\\ &= 16r^2-12rs-12sr+9s^2\\ &= 16r^2-24rs+9s^2.\end{aligned}$$

  This can be done faster if you remember eqn (2) above:

$$\begin{aligned}(4r-3s)^2 &= (4r)^2-2(4r)(3s)+(-3s)^2\\ &= 16r^2-24rs+9s^2.\end{aligned}$$

For some practice, please go to Exercise 2.

Exercise 1

1. Remove the brackets:

$a)\ 2(x-5)$ $b)\ 3(6-b)$ $c)\ (b+5)c$
$d)\ 2x(2a+3b)$ $e)\ -6(p-2q+4)$ $f)\ 3y(yz-2y+1)$

2. Expand and simplify:

$a)\ 3(x+1)+2(x+2)$ $b)\ 2(p-1)-(p-3)$ $c)\ 2(4m-3)-5(9-2m)$
$d)\ 2w(3w+1)+3w(2w-5)$ $e)\ 2a(a-1)-a(a-3)$ $f)\ 5j(3j+2)-2j(4j-1)$
$g)\ 3q(5q+4)+6(8-3q)$ $h)\ 2r(3r-1)-3r(3+2R)$.

3. Expand the following expressions and simplify where possible remembering the correct order of operations.

$a)\ 4(2m-3)+8$ $b)\ 9-3(4b+3)$ $c)\ 1-4(x-1)$
$d)\ 4-(5-2x)$ $e)\ 8m-3(1-2m)+6$ $f)\ 9c(4-c)+2(c-7)$.

Answers

1. $a)\ 2x-10$ $b)\ 18-3b$ $c)\ bc+5c$ $d)\ 4ax+6bx$ $e)\ -6p+12q-24$ $f)\ 3y^2z-6y^2+3y$
2. $a)\ 5x+7$ $b)\ p+1$ $c)\ 18m-51$ $d)\ 12w^2-13w$
   $e)\ a^{2.+a}$ $f)\ 7j^2+12j$ $g) 15q^2-6q+48$ $h)\ -11r$.
3. $a)\ 8m-4$ $b)\ -12b$ $c)\ 5-4x$ $d)\ -1+2x$ $e)\ 14m+3$ $f)\ 38c-9c^2-14$.

Exercise 2

1. Expand the following binomial products:

$a)\ (x+5)(x+3)$ $b)\ (u-5)(u+3)$ $c)\ (k+6)(j+1)$
$d)\ (y+2)(y-2)$ $e)\ (e-7)(e-8)$ $f)\ (t-3)(5-t)$
$g)\ (3q+2)(q+1)$ $h)\ (8a+5)(a-3)$ $i)\ (2c-d)(3c+2d)$
$j)\ (3a-2b)(3a+2b)$.

2. Remove the brackets:
$a)\ (d+3)^2$ $b)\ (x-2)^2$ $c)\ (3v+2)^2$
$d)\ (y-3z)^2$ $e)\ (3-2q)^2$ $f)\ (4f-3gh)^2$.

Answers

1. $a)\ x^2+8x+15$ $b)\ u^2-2u-15$ $c)\ kj+6j+k+6$ $d)\ y^2-4$ $e)\ e^2-15e+56$
   $f)\ -t^2-15+8t$ $g)\ 3q^2+5q+2$ $h)\ 8a^2-19a-15$ $i)\ 6c^2+cd-2d^2$ $j)\ 9a^2-4b^2$.
2. $a)\ d^2+6d+9$ $b)\ x^2-4x+4$ $c)\ 9v^2+12v+4$ $d)\ y^2-6yz+9z^2$ $e)\ 9-12q+4q^2$
   $f)\ 16f^2-24fgh+9g^2h^2$.