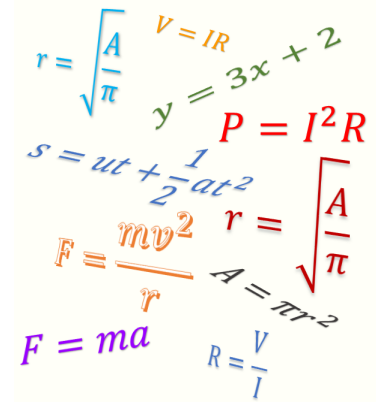


## A1.2 Algebraic Substitution

In many courses you will be required to use formulae to calculate something of interest. The process of substituting numbers for pro-numerals in an expression or formula is called substitution. This module provides some examples.

View a short video on substitution.



### Pro-numerals

Pro-numerals are letters or symbols that represent numbers in a mathematical expression or formula. In the expression

$$\frac{a + b}{2}$$

$a$  and  $b$  are pro-numerals. In the formula for the area of a circle<sup>1</sup>

$$A = \pi r^2$$

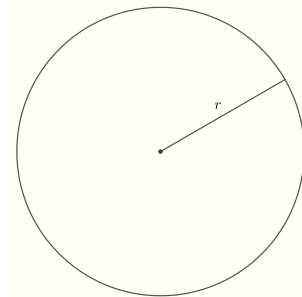
$A$ , the Greek letter  $\pi$  (pronounced pi) and the letter  $r$  are pro-numerals.

### Substitution

Putting a number into an expression or formula in place of pro-numerals is called substitution. For example if  $a = 5$  and  $b = 3$ , the expression  $\frac{a+b}{2}$  can be evaluated by substituting 5 for  $a$  and 3 for  $b$  wherever they occur. That is

$$\begin{aligned} \frac{a + b}{2} &= \frac{5 + 3}{2} \\ &= \frac{8}{2} \\ &= 4. \end{aligned}$$

<sup>1</sup>  $A$  is the area,  $r$  is the radius and  $\pi$  is a number that is approximately 3.14.



Similarly, the area of a circle with a radius  $r = 5 \text{ cm}$  is given by

$$\begin{aligned} A &= \pi r^2 \\ &= \pi 5^2 \\ &= 25\pi \\ &\approx 78.54 \text{ cm}. \end{aligned}$$

*Example 1*

Evaluate  $\frac{a+5}{b}$  if  $a = -9$  and  $b = 2$ .

**Solution:**

$$\begin{aligned} \frac{a+5}{b} &= \frac{-9+5}{2} \\ &= \frac{-4}{2} \\ &= -2. \end{aligned}$$

*Example 2*

Evaluate  $w^2 - 2z$  if  $w = -1$  and  $z = 5$ .

**Solution:**

$$\begin{aligned} w^2 - 2z &= (-1) \times (-1) - 2 \times 5 \\ &= 1 - 10 \\ &= -9. \end{aligned}$$

*Example 3*

Use the formula

$$C = \frac{5(F - 32)}{9}$$

to convert a temperature of  $212^\circ$ Fahrenheit (F) to Centigrade (C).

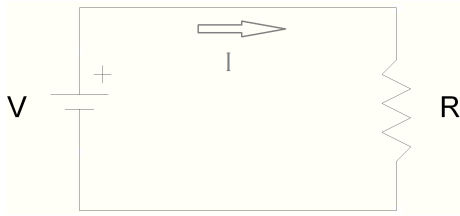
**Solution:**

Substituting  $F = 212$  into the formula gives:

$$\begin{aligned} C &= \frac{5(F - 32)}{9} \\ &= \frac{5(212 - 32)}{9} \\ &= \frac{5 \times 180}{9} \\ &= 100^\circ \text{ centigrade.} \end{aligned}$$

*Example 4*

The current in an electrical circuit is given by  $V = IR$  where  $V$  is the voltage (in volts),  $I$  is the current (in amps) and  $R$  is the resistance (in ohms).



If the resistance is 5 ohms and the current is 2 amps, what is the voltage,  $V$ ?

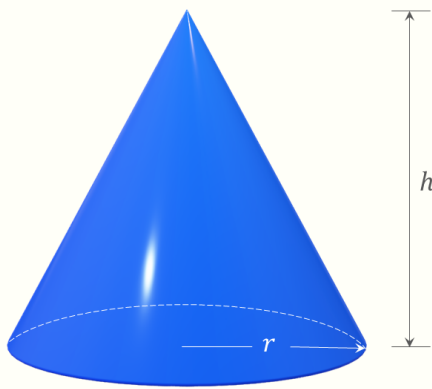
**Solution:**

We have  $r = 5$  and  $I = 2$ . Therefore, the voltage

$$\begin{aligned} V &= IR \\ &= 2 \times 5 \\ &= 10 \text{ volts.} \end{aligned}$$

*Example 5*

The volume  $V$ , of a right circular cone with base radius  $r$  and height  $h$  is given by  $V = \frac{1}{3}\pi r^2 h$ .



If the radius of a cone is 5 cm and its height is 15 cm. what is the volume of the cone?

**Solution:**

In this case  $r = 5$  and  $h = 15$ , so the volume of the cone is

$$\begin{aligned} V &= \frac{1}{3}\pi r^2 h \\ &= \frac{1}{3} \times \pi \times 5^2 \times 15 \\ &\approx 392.7 \text{ cm}^3. \end{aligned}$$

*Example 6*

The formula relating distance traveled  $s$ , to initial speed  $u$ , acceleration  $a$  and time  $t$  is

$$s = ut + \frac{1}{2}at^2.$$

A car traveling at a speed of  $4 \text{ m/s}$  accelerates at a rate of  $2 \text{ m/s}^2$  for  $5 \text{ s}$ . How far does it travel during this time?

**Solution:**

In this case,  $u = 4$ ,  $a = 2$  and  $t = 5$ . So the distance traveled is

$$\begin{aligned} s &= ut + \frac{1}{2}at^2 \\ &= 4 \times 5 + \frac{1}{2} \times 2 \times 5^2 \\ &= 20 + \frac{1}{2} \times 2 \times 25 \\ &= 45 \text{ m.} \end{aligned}$$

*Example 7*

The area of a circle  $A$  is given by

$$A = \pi r^2$$

where  $r$  is the radius. If a circle has an area of  $30 \text{ cm}^2$  what is its radius.

**Solution:**

Substituting  $A = 30$  we have

$$30 = \pi r^2.$$

We want  $r$ . First we divide both sides by  $\pi$  to get

$$\begin{aligned} \frac{30}{\pi} &= \frac{\pi r^2}{\pi} \\ &= r^2 \\ r^2 &= \frac{30}{\pi} \\ r &= \sqrt{\frac{30}{\pi}} \\ &\approx 3.1 \text{ cm.} \end{aligned}$$

*Exercises*

1. Evaluate the following:

- |                                  |  |
|----------------------------------|--|
| a) $-4k$ if $k = 7$              | b) $2mn$ if $m = 4$ , $n = -2$           |
| c) $e^2 - 5$ if $e = 2$          | d) $5 - b - b^2$ if $b = 3$              |
| e) $2k^2 + 4$ if $k = -6$        | f) $-3ab^2$ if $a = 4$ , $b = 2$         |
| g) $\frac{n}{4} + 2$ if $n = 10$ | h) $\frac{u}{5v}$ if $u = -20$ , $v = 2$ |

*Answers*

a)  $-28$  b)  $-16$  c)  $-1$  d)  $11$  e)  $76$  f)  $-48$  g)  $4.5$  h)  $-2$   
2. Evaluate the following if  $a = -1$ ,  $b = 6$ ,  $c = 3$ ,  $m = -2$ ,  $n = 2$ .

a)  $3a^2 - 7$       b)  $3a - b^2$   
c)  $(2m + 1)^2$       d)  $3(a - b^2)$   
e)  $\frac{2m}{n}$       f)  $(m - n)^2$

*Answers*

a)  $-4$  b)  $-39$  c)  $9$  d)  $15$  e)  $-2$  f)  $16$