RMIT UNIVERSITY

A1.1 Algebraic Operations

In this module we introduce the basic skills for addition, subtraction, multiplication and division of algebraic expressions. Courses in science, engineering and other fields require you to have these skills.

Play a short video on Algebraic Operations

Download a script of the video.

A page from Al-Khwarizmi's "al-Kitab al-muhtasar fi hisab al-gabr wa-l-muqabala", an early book on algebra. (Image from https://en.wikipedia.org/wiki/The_Compendious_Book_on_Calculation_by_Completion_and_Balancing)

Like and Unlike Terms

Like terms contain exactly the same pro-numerals[1]. The following table gives some examples of like and unlike terms.

Like Terms	Unlike Terms
$3x, 5x$	$3x, 5y$
$2a, -3a$	$3a, 3$
$m^2, 7m^2$	$m^2, 7m$
$2ab, 3ab$	$2a^2b, 3ab^2$
$3xyz, 5xyz$	$3xyz, 3xy$
$3ef, 3fe$	$3ef, 3eg$
$6\alpha\beta$, $\beta\alpha$	$6\alpha\beta\gamma$, $6\alpha\beta$

For some practice, please go to Exercise 1 below.

[1] A pro-numeral is a combination of letters, symbols and numbers. For example, $3x$ is a pro-numeral. In algebra, when we write $3x$ we really mean $3 \times x$. Another example of a pro-numeral is $5ab$. This means $5 \times a \times b$. Note that $5ab = 5ba$. The order of the letters and numbers is not important but usually we write numbers first and use alphabetical order for the letters. While $5ba$ is correct, it is more common to write $5ab$.

Addition and Subtraction

The essential rule is:

Only like terms may be added or subtracted.

Example 1.

$$\begin{aligned} 7e + 10e &= 7 \times e + 10 \times e \\ &= (7 + 10)\, e \\ &= 17e. \end{aligned}$$

In practice we would not put in the first line. It is included to reinforce what $7e$ and $10e$ mean. In practice we would probably go straight to the answer. That is $7e + 10e = 17e$.

Example 2.

$$\begin{aligned} 3x^2 - x^2 - 4x^2 &= (3 - 1 - 4)\, x^2 \\ &= -2x^2. \end{aligned}$$

Example 3. [2]

$$\begin{aligned} 3m - 4n + 6m + n &= (3 + 6)\, m + (-4 + 1)\, n \\ &= 9m - 3n. \end{aligned}$$

[2] This example shows how we can gather different like terms. In this case m and n.

Example 4.

$$\begin{aligned} 3a - b - 5a + 4ab - 3b + ab &= (3 - 5)\, a + (-1 - 3)\, b + (4 + 1)\, ab \\ &= -2a - 4b + 5ab. \end{aligned}$$

Example 5.

$3x - x^2$ there are no like terms so nothing can be done.

Example 6.

$$p + 2p - 3 = 3p - 3.$$

Example 7.

$$8uv + 3u - 10vu = -2uv + 3u.$$

Example 8.

$6r^2s - 2rs^2$ there are no like terms so nothing can be done.

For some practice, please go to Exercise 2 below.

Multiplication

Both like and unlike terms may be multiplied. When multiplying two or more terms consider:

1. The sign of the answer.[3]

2. The product of the terms.

[3] In doing the multiplication, remember the basic rules for determining the sign of the product.

$$\begin{aligned} (+ve) \times (+ve) &= (+ve) \\ (+ve) \times (-ve) &= (-ve) \\ (-ve) \times (+ve) &= (-ve) \\ (-ve) \times (-ve) &= (+ve)\,. \end{aligned}$$

Here are some examples:

1. $(-4) \times (-3b) = 12b$
2. $-2 \times 6y = -12y$
3. $2e \times \left(-5e^2\right) = -10e^3$
4. $\left(-2u^2v\right) \times (-4v) = 8u^2v^2$
5. $-3pq \times (-2q) \times p = 6p^2q^2$

Division

In algebra, we often use fractions. For example:[4]

$$\frac{2}{5} = 2 \div 5.$$

Similarly for algebraic terms:

$$\frac{2x^2yz}{3xy^2} = 2x^2yz \div 3xy^2.$$

When dividing algebraic terms:

1. Rewrite as a fraction if necessary
2. Expand any powers
3. Establish the sign of the answer[5]
4. Cancel any common factors

Examples

1. Divide $-12wyz$ by $3yz$.
   Solution:

$$\begin{aligned} -12wyz \div 3yz &= \frac{-12wyz}{3yz} \quad \text{write as a fraction} \\ &= -\frac{12wyz}{3yz} \quad \text{determine sign} \\ &= -4w \quad \text{cancel common factors.} \end{aligned}$$

2. Divide $2m^2n$ by $-6mn^2$.
   Solution:

$$\begin{aligned} 2m^2n \div 6mn^2 &= \frac{2m^2n}{-6mn^2} \\ &= -\frac{2m^2n}{6mn^2} \\ &= -\frac{m}{3n}. \end{aligned}$$

3. Simplify $6a^2 \times 4ab \div 12ab$.
   Solution:

$$\begin{aligned} 6a^2 \times 4ab \div 12ab &= \frac{6a^2 \times 4ab}{12ab} \quad \text{write as a fraction} \\ &= \frac{24a^3b}{12ab} \quad \text{establish sign and expand powers} \\ &= 2a^2 \quad \text{cancel common factors.} \end{aligned}$$

[4] In this case the top term, 2 is called the numerator, the bottom term 5, is called the denominator and the horizontal line between them is called the fraction bar, division bar or vinculum.

[5] In doing the division, remember the basic rules for determining the sign of the quotient.

$$\begin{aligned} (+ve) \div (+ve) &= (+ve) \\ (+ve) \div (-ve) &= (-ve) \\ (-ve) \div (+ve) &= (-ve) \\ (-ve) \div (-ve) &= (+ve)\,. \end{aligned}$$

4. Divide $m + 2$ by $4m$

$$(m+2) \div 4m = \frac{m+2}{4m}.$$

There are no common factors and so it is not possible to simplify further.

For practice examples see Exercise 3.

Order of Operations

The basic operations of arithmetic are multiplication, division, addition and subtraction. Usually the order in which we perform the operations is important.

For example, what is the answer to the following arithmetic problem

$$4 + 3 \times 2 + 5\ ?$$

At first sight there are several possible answers.

Working from left to right we get:

$$\begin{aligned} 4 + 3 \times 2 + 5 &= 7 \times 2 + 5 \\ &= 14 + 5 \\ &= 19. \end{aligned}$$

Working from right to left we get:

$$\begin{aligned} 4 + 3 \times 2 + 5 &= 4 + 3 \times 7 \\ &= 4 + 21 \\ &= 25. \end{aligned}$$

If we do the multiplication first we get another answer:

$$\begin{aligned} 4 + 3 \times 2 + 5 &= 4 + 6 + 5 \\ &= 15. \end{aligned}$$

Yet another answer is obtained by doing the additions first:

$$\begin{aligned} 4 + 3 \times 2 + 5 &= 7 \times 7 \\ &= 49. \end{aligned}$$

Mathematicians don't like imprecision. In order to allow only one answer to the problem we perform operations in the following order:

1. **B**rackets

2. **I**ndices

3. **M**ultiplication and **D**ivision from left to right
4. **Addition** and **S**ubtraction

Using the first letters (bold) we get the abbreviation **BIMDAS.**

Applying this rule the answer to our problem is

$$\begin{aligned} 4+3\times 2+5 &= 4+6+5 \quad \text{multiplication first} \\ &= 15 \quad \text{then addition.} \end{aligned}$$

Examples:

1. $3\times 2+4=6+4=10.$
2. $3+2\times 4=3+8=11.$
3. $(3+2)\times 4=5\times 4=20.$
4. $3-2^2=3-2\times 2=3-4=-1.$
5. $(3-2)^2=(3-2)\times(3-2)=1\times 1=1.$
6. $3^2-2^2=3\times 3-2\times 2=9-4=5.$
7. $3st-3s\times 4t=3st-12st=-9st.$

For some practice, see Exercise 4.

Exercise 1

Which of the five terms on the right is a like term with the term on the left?

1)	$3x$	3	$2x$	$3x^2$	$2xy$	$4x^2a$
2)	$2ab$	$2a$	$2b$	$3x^2$	$6abc$	$12ab$
3)	$2x^2$	x	$2x$	$5x^2$	$4x$	$4x^3$
4)	$3xy^2$	3	$3xy$	$3x^2xy^2$	$3y^2$	y^2x
5)	$2m^2n$	n	mn^2	$8mn$	$2m^2$	$4nm^2$
6)	$4ab^2c$	4	$2ab^2c$	$4abc$	$8b^2c$	$4cba$

Exercise 2

Simplify each of the following:

a) $13x+4-3x-1$	*b*) $10mn+5m+12mn+6m$
c) $3xy^2+2xy+5xy^2+3xy$	*d*) $5xy+6m-2xy-2m$
e) $x+y+2x-y$	*f*) $3a+5b-a-6b$
g) $x+4y-x-2y$	*h*) $7x-4m-5x-3m$
i) $4x-5x-3y+5x$	*j*) $9mn-3m-n+4m^2$

Exercise 3

Simplify the following algebraic expressions:

1.
a) $5 \times 2k$ *b*) $4a \times 3ab$
c) $y \times 3y$ *d*) $4m \times (-3mn)$
e) $m \times 39 \times 5$ *f*) $2ab \times 3bc \times (-4)$
g) $2ab^2 \times 3ac$ *h*) $4m \times (-5kmp)$

2.
a) $18ef \div 6f$ *b*) $-100uvw \div 100w$
c) $24gh^2 \div 8gh$ *d*) $3m^2n \div 12mn^2$
e) $rs \times 2st \div 2s$ *f*) $3jk \times 12km \div 9jkm$
g) $10p \times 3qp \div 16pq$ *h*) $4yz \times 5w^2z \div 10wy$

Exercise 4

Simplify the following expressions:

1) $18 + (3 \times (-5))$ 2) $3 \times (-4) + (8 \times 2)$
3) $10 - 5^2 + 3$ 4) $(10-5)^2 + 3$
5) $10^2 - 5^2$ 6) $(10-5)^2$
7) $3m + 2 \times 3m$ 8) $6ab - 3a \times 4b$
9) $16gh - 4gh \times 4$ 10) $9b - 3b \times 2k + 2k \times b$

Answers

Exercise 1

1. $2x$ 2. $12ab$ 3. $5x^2$ 4. y^2x 5. $4nm^2$ 6. $2ab^2c$

Exercise 2

a) $10x+3$ *b*) $22mn + 11m$ *c*) $8xy^2 + 5xy$ *d*) $3xy + 4m$ *e*) $3x$
f) $2a - b$ *g*) $2y$ *h*) $2x - 7m$ *i*) $5x - 3y$ *j*) $9mn - 3m - n + 4m^2$

Exercise 3

1. *a*) $10k$ *b*) $12a^2b$ *c*) $3y^2$ *d*) $-12m^2n$
e) $15mp$ *f*) $-24ab^2c$ *g*) $6a^2b^2c$ *h*) $-20km^2p$

2. *a*) $3e$ *b*) $-uv$ *c*) $3h$ *d*) $m/4n$
e) rst *f*) $4k$ *g*) $15p/8$ *h*) $2wz^2$

Exercise 4

1. 3 2. 4 3. -12 4. 28 5. 75
6. 25 7. $9m$ 8. $-6ab$ 9. 0 10. $9b - 4bk$