IN4 DEFINITE INTEGRALS

$\int_a^b f(x)\,dx$ is called the definite integral from x = a to x = b where a is the lower limit of integration and b is the upper limit of integration.

$$\int_a^b f(x)\,dx = [F(x)]_a^b = F(b) - F(a) \text{ where } F(x) = \int f(x)\,dx$$

This can be calculated only if $f(x)$ is defined for all x in the interval a ≤ x ≤ b

Examples

1. $\int_2^4 (x+1)\,dx = \left[\frac{x^2}{2} + x\right]_2^4$

$= \left[\frac{4^2}{2} + 4\right] - \left[\frac{2^2}{2} + 2\right]$

$= 8$

2. $\int_0^\pi (cosx + e^{-2x})\,dx = \left[sinx - \frac{e^{-2x}}{2}\right]_0^\pi$

$= \left[sin\,\pi - \frac{e^{-2\pi}}{2}\right] - \left[sin\,0 - \frac{e^{-2(0)}}{2}\right]$

$= \left[0 - \frac{e^{-2\pi}}{2}\right] - \left[0 - \frac{1}{2}\right]$

$= \frac{1 - e^{-2\pi}}{2}$

3. *If the work done (measured in joules) in moving an object from point a to point b is given by* $W = \int_a^b (3x^2 + 2)\,dx$ *find the work done in moving the object from the point* $x = 0$ *to the point* $x = 3$.

$\int_0^3 (3x^2 + 2)\,dx = \left[\frac{3x^3}{3} + 2x\right]_0^3$

$= \left[\frac{3(3)^3}{3} + 2(3)\right] - \left[\frac{3(0)^3}{3} + 2(0)\right]$

$= (3^3 + 6) - (0)$

$= 33$ joules

Exercises

1. Evaluate exactly:

   (a) $\int_0^2 (3x^2 + x + 1)\, dx$ (b) $\int_0^{\pi} (\cos x + sin2x)\, dx$ (c) $\int_{-2}^{4} 2e^{-3x}\, dx$

2. The acceleration of a particle is given by $a(t) = 2t^2 + 3e^{-t}$ m/s^2. If its initial velocity, $v(0)$, is 2 m/s find the velocity when $t = 3$. [NB: acceleration $a(t) = v'(t)$]

Answers

1(a) 12 (b) 0 (c) $\frac{2e^6}{3} - \frac{2e^{-12}}{3}$

2. 22.88m/s